Inertial effect in uniform ferromagnets

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THE SLOWEST
FAST-DEGREES-OF-FREEDOM

GdR Meeticc

Chédigny

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W. Pauli and N. Bohr looking at a spinning top.

Outline

I) Beyond the precession of the magnetization: the inertial regime and nutation.

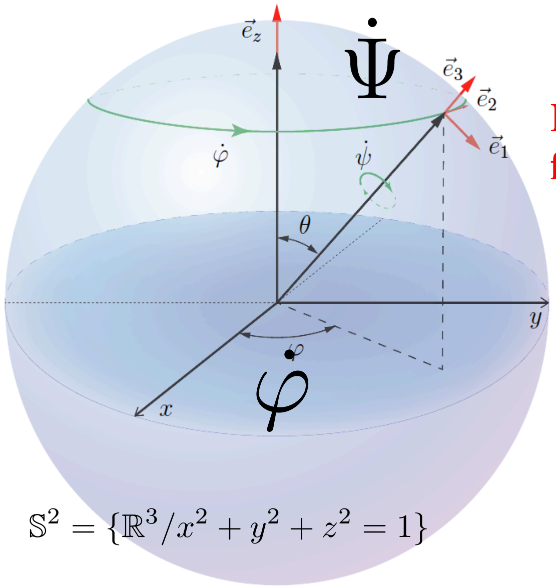
II) The Ampere electrodynamic modele

III) Geometrical phase and magnetic monopole

IV) Conclusion

The Gilbert approach (rigid rotator)

Mechanical analogy with the spinning stick: $\vec{M} = M_s \vec{e}_3$



$\mathbb{S}^2 = \{\mathbb{R}^3 / x^2 + y^2 + z^2 = 1\}$

Configuratione space

Rotating frame

Angular velocity

$$\Omega_1 = \dot{\varphi}\,\sin\theta\,\sin\psi + \dot{\theta}\,\cos\psi,$$
$$\Omega_2 = \dot{\varphi}\,\sin\theta\,\cos\psi - \dot{\theta}\,\sin\psi,$$
$$\Omega_3 = \dot{\varphi}\,\cos\theta\ + \dot{\psi}.$$

Inertial tensor

$$\bar{I} = \begin{pmatrix} I_1 & 0 & 0 \\ 0 & I_1 & 0 \\ 0 & 0 & I_3 \end{pmatrix}$$

Newton

$$\frac{d\vec{M}}{dt} = \vec{\Omega} \times \vec{M}$$

Angular momentum

$$\vec{L} = \bar{I}\vec{\Omega}$$

$$\vec{\Omega} = \frac{\vec{M}}{M_s^2} \times \frac{d\vec{M}}{dt} + \Omega_3 \vec{e}_3$$

=>

$$\vec{L} = \frac{I_1}{M_s^2}\left(\vec{M} \times \frac{d\vec{M}}{dt}\right) + \frac{\vec{M}}{\gamma} \quad => \quad \vec{L} = \frac{I_1}{M_s^2}\left(\vec{M} \times \frac{d\vec{M}}{dt}\right) + L_3 \vec{e}_3$$

Without loss of generality:

$$M_s = \gamma L_3 = \gamma I_3 \Omega_3$$

Lagrange

Introducing damping: Lagrange approach:

Kinetic energy *Potential energy*

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_i} - \frac{\partial \mathcal{L}}{\partial q_i} + \frac{\partial \mathcal{F}}{\partial \dot{q}_i} = 0$$

where

$$\mathcal{L} = \frac{1}{2}\left(I_1(\Omega_1^2 + \Omega_2^2) + I_3\Omega_3^2\right) - V(\theta, \varphi)$$

Rayleigh-Gilbert dissipative function:

$$\mathcal{F} = \frac{\eta}{2}\left(\frac{dM}{dt}\right)^2 = \frac{1}{2}\eta M_s^2(\Omega_1^2 + \Omega_2^2)$$

Rayleigh

The Gilbert equation

Killing inertia: $d\Omega/dt = 0$

$$\boxed{I_1 = I_2 = 0}$$

A) Gilbert's ad hoc assumption: the gyromagnetic relation

$$\vec{L} = \frac{I_1}{M_s^2}\cancel{\left(\vec{M} \times \frac{d\vec{M}}{dt}\right)} + \frac{\vec{M}}{\gamma}$$

Gyromagnetic relation recovered!

- T. L. Gilbert
  PhD dissertation 1956 (appendix B)
- T. L. Gilbert
  Phys. Rev **100**, 1243 1955 **(abstract)**
- T. L. Gilbert IEEE Trans Mag **40**, 2004
  (paper: postdeadline submission?)

$\Longrightarrow$ Reduction of the phase space to the configuration space

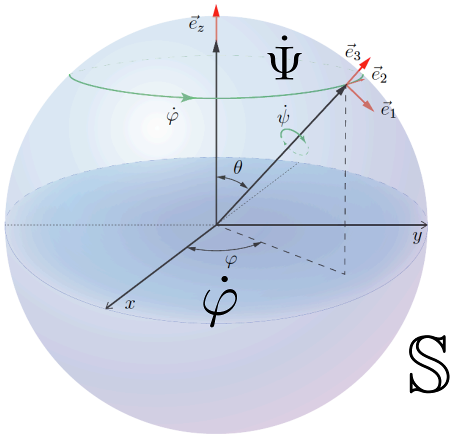
In a case of the analogy with ponctual Brownian particule :
From Klein-Kramers equation to diffusion equation

Back to Lagrange equation (damping):

LLG equation = kinetic equation

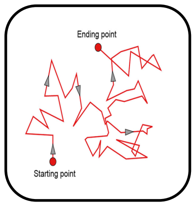
$$\vec{\Omega}_G = \begin{Bmatrix} \dot{\theta} \\ \dot{\varphi}\, sin\theta \\ \dot{\varphi}\, cos\theta + \dot{\psi} \end{Bmatrix} = \begin{Bmatrix} \frac{-\gamma}{M_s sin\theta}\frac{\partial V}{\partial \varphi} + \gamma\eta M_s \dot{\varphi}\, sin\theta \\ \frac{-\gamma}{M_s}\frac{\partial V}{\partial \theta} - \gamma\eta M_s \dot{\theta} \\ \frac{M_s}{\gamma I_3} \end{Bmatrix} \Rightarrow \frac{d\vec{M}}{dt} = \boxed{\gamma \vec{M} \times \left(-\frac{\partial V}{\partial \vec{M}} - \eta \frac{d\vec{M}}{dt}\right)}$$

$$\frac{d\vec{M}}{dt} = \vec{\Omega} \times \vec{M}$$



$\mathbb{S}^2$

Brownian motion
at equilibrium



Force = effective magnetic fied:

$$\vec{H} = -\vec{\nabla}_\Sigma V \equiv -\frac{\partial V}{\partial \vec{M}}$$

LLG equation is very robust but $I_1 = I_2 = 0$ is unphysical (I)

Gilbert, footnote 7 in IEEE Trans. Mag 40 (2004):
« I was unable to conceive of a physical object with an inertial tensor of this kind. »

Brown Am. J. Phys. 1960 p 549
« We treat the rotating moment system as a symmetric top, with principal moment of inertia $I_1=I_2=0$, $I_3>0$. For a top made of classical mass particle $I_1=I_2=0$ implies $I_3=0$; but this top is not made of classical mass particles ».

Morrish textbook p 551 **(**1980**) :** many editions from 1965 to 2002 :
«A Lagrangian function *L* consistent with the accepted equation of motion d**M**/dt = g **M** x **H** + **damping** can be obtained by considering the magnetic system as a classical top with principal momet of inertia $(0,0,I_3)$».

... other textbooks citations in J.-E. Wegrowe, M.-C. Ciornei Am. J. Phys. **80**, 607 (2012)

Conclusion: Why does the model give the good result?

LLG equation is very robust but

$I_1 = I_2 = 0$ is unphysical (I)

Conclusion:

Why does the model give the correct result?

The Gilbert's assumption is actually the separation between the fast and slow degrees of freedom.

$$\dot{\Psi} \gg \dot{\varphi} cos\theta$$

belongs to the environment: reduction into the damping constant and noise.

… as shown in the next section

Geometrical phase: efficient tool for the analysis of this separation

LLG equation with inertia

Beyond Gilbert's assumption : non-zero inertia $I_1 = I_2 \neq 0$

$$\vec{H} = H\vec{e}_z$$

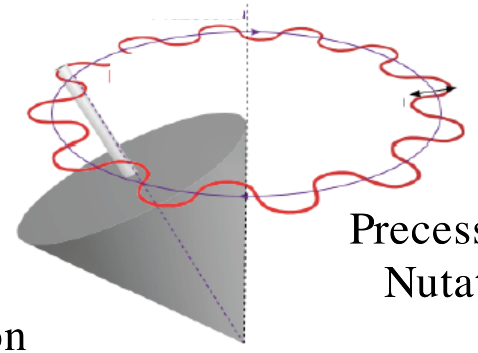
⇔ If Fast $\dot{\Psi}$ and slow $\dot{\varphi}$ time scales are of the same order

Non-dissipative case (back to slide 3):

$$\vec{L} = \frac{I_1}{M_s^2}\left(\vec{M} \times \frac{d\vec{M}}{dt}\right) + \frac{\vec{M}}{\gamma}$$

+ Newton law

$$\frac{d\vec{L}}{dt} = \vec{M} \times \vec{H}$$



Precession + Nutation

$$\Rightarrow \boxed{\frac{d\vec{M}}{dt} = \gamma\vec{M} \times \left(\vec{H} - \frac{I_1}{M_s^2}\frac{d^2\vec{M}}{dt^2}\right)}$$

Dynamics of the magnetization

Nutation oscillations superimposed to precession

Dissipative case: generalized LLG equation:

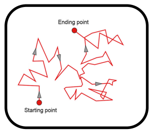
$$\Rightarrow \boxed{\frac{d\vec{M}}{dt} = \gamma\vec{M} \times \left[\vec{H} - \eta\left(\frac{d\vec{M}}{dt} + \tau\frac{d^2\vec{M}}{dt^2}\right)\right]}$$

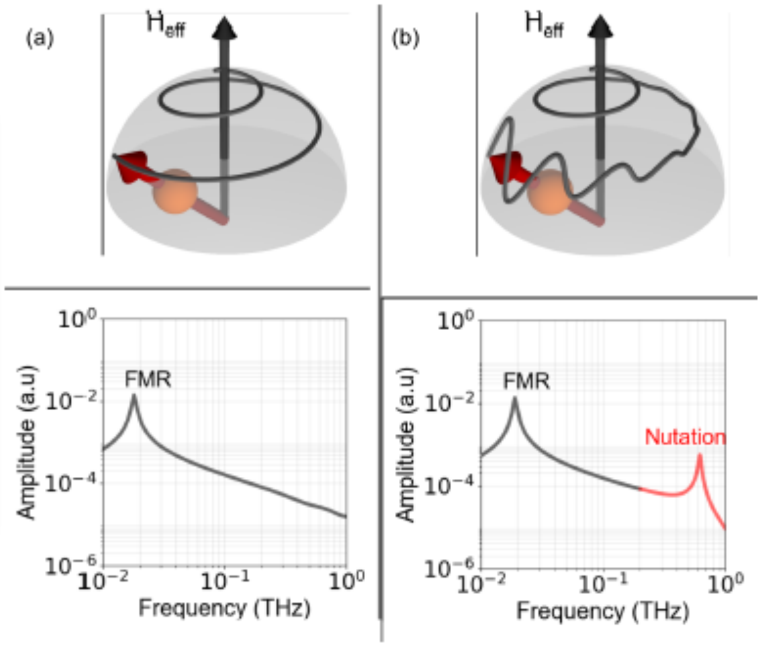
Fast relaxation time of the Angular momentum ***L*** *(like for Brownian motion)*

$$\tau \equiv \frac{I_1}{\eta M_s^2}$$

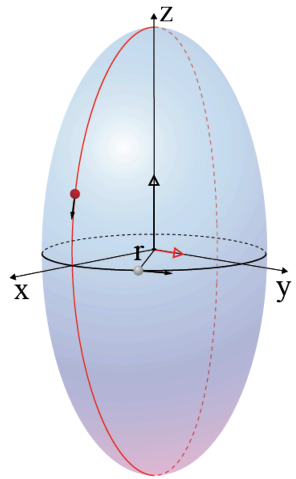
Diffusion in the Phase space

$$\mathbb{S}^2 \otimes \mathbb{R}^3$$





Beyond Gilbert ad-hoc assumption: the spining top



Lagrange Equations :

$$\begin{cases} \frac{d}{dt}[I_1\dot\theta] & = I_1\dot\varphi^2 \sin\theta\cos\theta - I_3\dot\varphi\sin\theta\left(\dot\varphi\cos\theta+\dot\psi\right) - \frac{\partial\mathcal{F}}{\partial\dot\theta} - \frac{\partial V}{\partial\theta} \\ \frac{d}{dt}\left[I_1\dot\varphi \sin^2\theta + I_3\left(\dot\varphi\cos\theta+\dot\psi\right)\cos\theta\right] & = -\frac{\partial\mathcal{F}}{\partial\dot\varphi} - \frac{\partial V}{\partial\varphi} \\ \frac{d}{dt}\left[\boxed{I_3\left(\dot\varphi\cos\theta+\dot\psi\right)}\right] & = -\cancel{\frac{\partial\mathcal{F}}{\partial\dot\psi}} = 0 \end{cases}$$

Spinning is non-dissip

$$L_3 = I_3\Omega_3 = \frac{M_s}{\gamma}$$

$$\begin{cases} \dot\Omega_1 & = -\frac{\Omega_1}{\tau} + \Omega_3\left(1-\frac{I_3}{I_1}\right)\Omega_2 - \frac{M_s}{I_1}H_2 \\ \dot\Omega_2 & = -\frac{\Omega_2}{\tau} - \Omega_3\left(1-\frac{I_3}{I_1}\right)\Omega_1 + \frac{M_s}{I_1}H_1 \\ \Omega_3 & = \frac{M_s}{\gamma I_3} \end{cases}$$

Where:

$$\frac{d\vec M}{dt} = \vec\Omega\times\vec M \qquad \vec\Omega = \frac{\vec M}{M_s^2}\times\frac{d\vec M}{dt} + \frac{\vec M}{I_3\gamma}. \qquad \dot{\vec\Omega}\cdot\vec e_3 = 0$$

<=>

$$\boxed{\frac{d\vec M}{dt} = \gamma\vec M\times\left[\vec H - \eta\left(\frac{d\vec M}{dt} + \tau\frac{d^2\vec M}{dt^2}\right)\right]}$$

with relaxation time

$$\tau \equiv \frac{I_1}{\eta M_s^2}$$

relaxation time for the fast degrees of freedom

J.-E. Wegrowe, M.-C. Ciornei Am. J. Phys. **80**, 607 (2012)

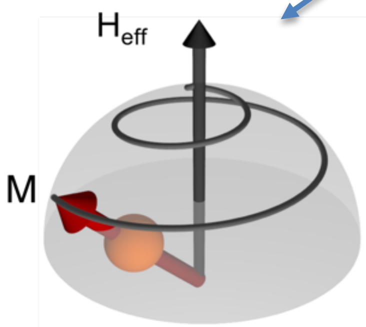
Frequency response (small AC field perpendicular to static field)

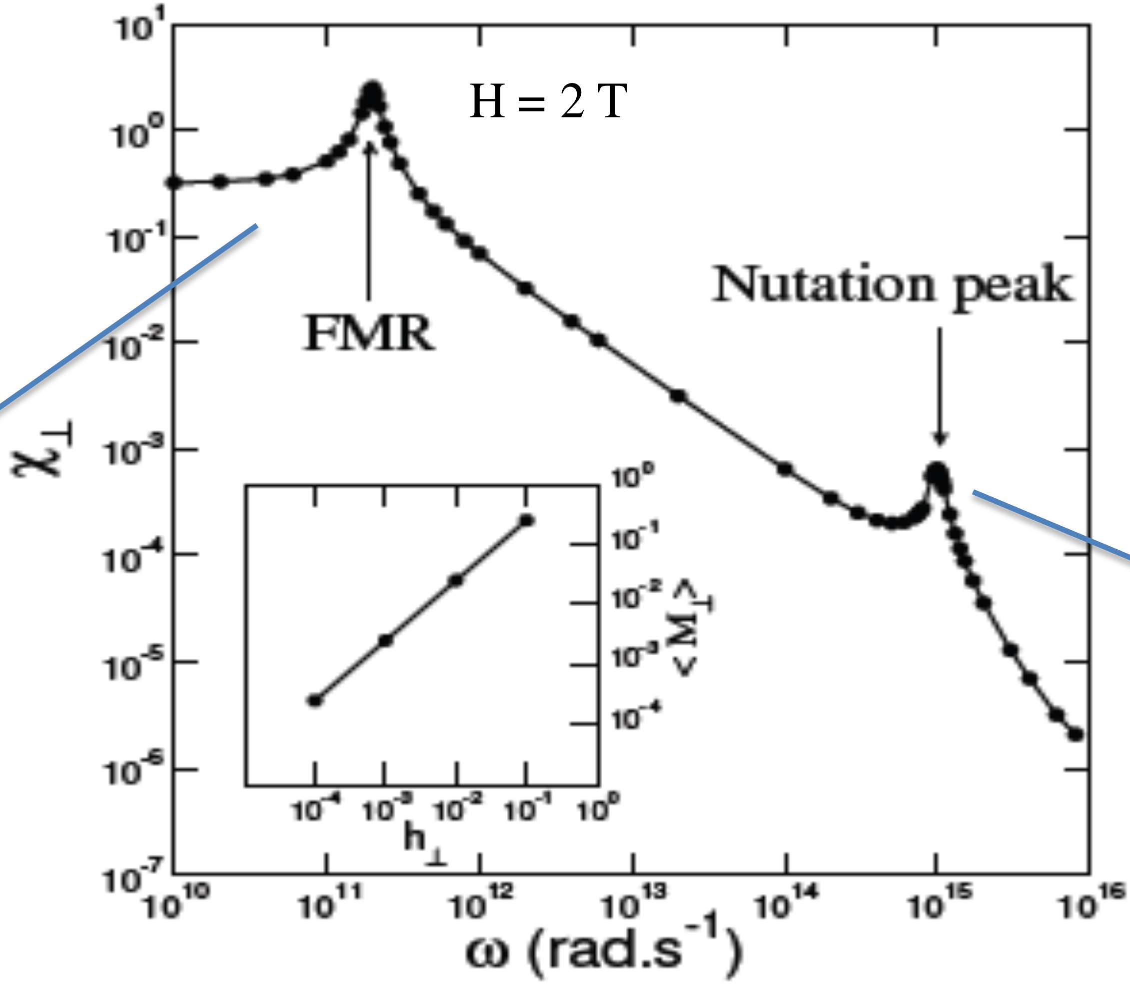
Numerical stud

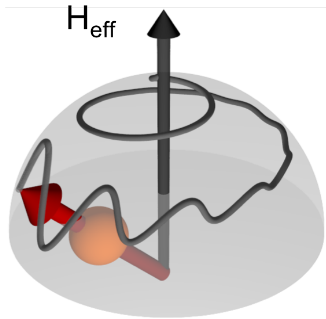
$\alpha = 0.1$

$\tau = 10^{-14}\ s$

$\tau = \frac{I_1}{I_3} \frac{1}{\alpha \Omega_3}$







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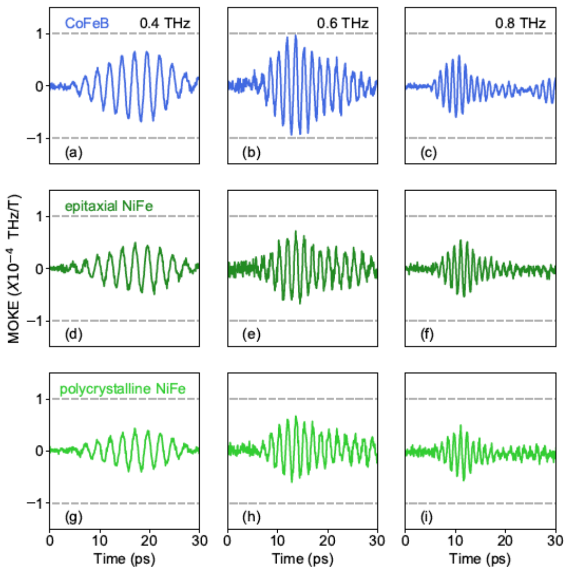
« I will show the experimental confirmation next year »
(from 2009 to 2018)

Experimental evidence 2019

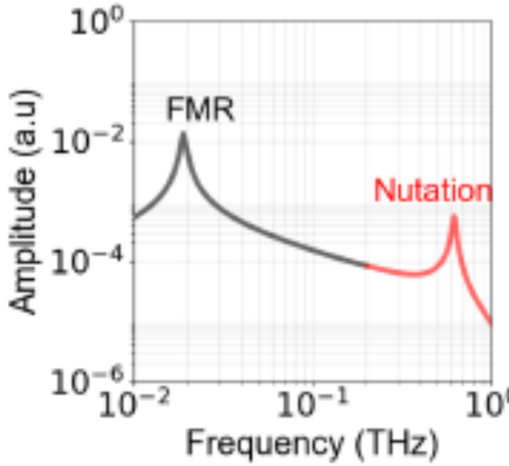
by Kumar Neeraj et al. under the supervision of Stefano Bonetti
at Helmoltz-Zentrum Dresden (FEL)

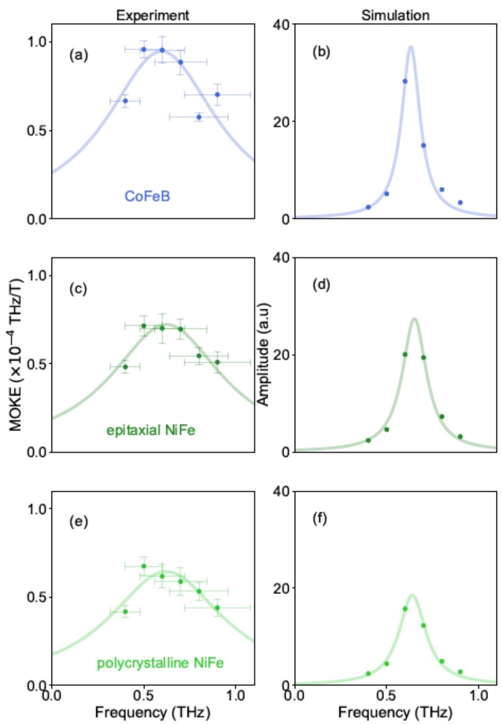
Experimental evidence of inertial dynamics in ferromagnets

Kumar Neeraj[1,†], Nilesh Awari[2,†], Sergey Kovalev[2], Debanjan Polley[1], Nanna Zhou Hagström[1], Sri Sai Phani Kanth Arekapudi[3], Anna Semisalova[4,5], Kilian Lenz[5], Bertram Green[2], Jan-Christoph Deinert[2], Igor Ilyakov[2], Min Chen[2], Mohammed Bowatna[2], Valentino Scalera[6], Massimiliano D'Aquino[7], Claudio Serpico[6], Olav Hellwig[3,5], Jean-Eric Wegrowe[8], Michael Gensch[9,10], and Stefano Bonetti *[1,11]



Peak at 0.6 THz





CoFeB

NiFe
Single crystal

NiFe
Polycrystalline

Ten years and different approaches

- M.-C. Ciornei, M. Rubi, J.-E. Wegrowe, *Magnetization dynamics in the inertial regime: nutation predicted at short time scales*, Phys. Rev. B **83**, 020410(R) (2011) ,
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- D. Thonig, O. Eriksson and M. Pereiro, *Magnetic moment of inertia within the torque-torque correlation model*, Scientific reports DOI :10.1038/s41598-017-01081-z (2017).
- **R. Mondal, M. Berritta, A. K. Nandy, P. M. Oppenneer, *Relativistic theory of magnetic inertia in ultrafast spin dynamics*, Phys. Rev. B 96, 024425 (2017)**
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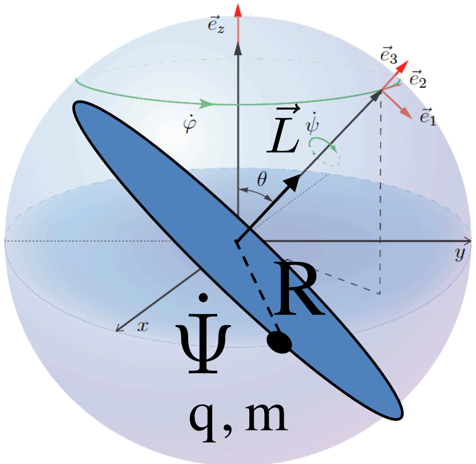
The Ampère static dipole I:

*<u>Electrodynamical model of the rotating charge q in a **loop**</u>*



Calculation of the <u>potential vector</u> $A_d(r)$ in the dipolar approximation:

$$\vec{A}_d(r) = \frac{\mu_0 \vec{M} \times \vec{r}}{4\pi r^3}$$ => **<u>Defines:</u>** $$\boxed{\vec{M} = \frac{qR^2\dot{\Psi}}{2}\vec{u}}$$

Magnetization

<u>The angular momentum is</u>:

$$\vec{L} = mR^2\dot{\Psi}\,\vec{u} \quad \Rightarrow \quad \boxed{\vec{M} = \gamma\vec{L}}$$

Loop of radius R at rest:

$I_3 = m R^2$

Gyromagnetic relation : where $\gamma = q/(2m)$

<u>Force = External magnetic fied</u>:

$$\vec{H} = -\vec{\nabla}_\Sigma V \equiv -\frac{\partial V}{\partial \vec{M}}$$

<u>Newton's law (torque)</u>:

$$d\vec{L}/dt = \vec{M} \times \vec{H}$$

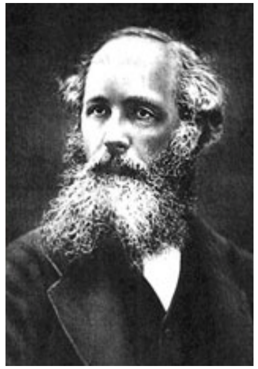
$$\boxed{d\vec{M}/dt = \gamma\vec{M} \times \vec{H}.}$$

<u>Dipole dynamics = moving loop</u>

assuming $$\vec{A}_d(r,t) = \frac{\mu_0\, M(t) \times \vec{r}}{4\pi r^3}$$

What happen if $\dot{\psi} \sim \varphi\, cos\theta$?

Ampère



Maxwell

Newton

The Ampère dipole II: the moving loop

Vector potential A(r,t) produced by a confined electric charge of velocity ***v*** and trajectory ***w*** at a distance ***r***.

D.J. Gridfiths, « Introduction to Electrodynamics:

$$\vec{A}(\vec{r},t) = \frac{\mu_0 q}{4\pi} \frac{\vec{v}(t_r)}{|\vec{r}-\vec{w}(t_r)| - \frac{1}{c}(\vec{r}-\vec{w}(t_r)).\vec{v}(t_r)}$$

retarded time t_r:

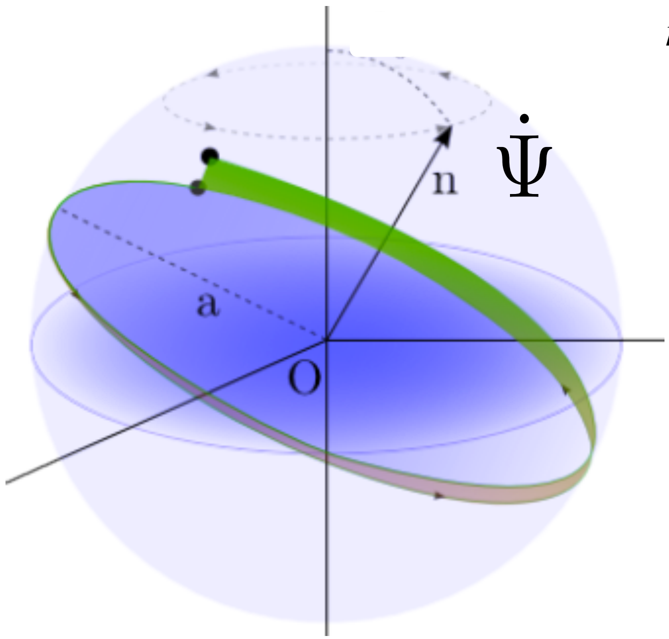
$$c(t-t_r) = |\vec{r}-\vec{w}(t_r)|$$

<u>*approximations:*</u>

non-relativistic $\vec{v}/c \ll 1$

non-radiative $|t-t_r| \ll \frac{r}{c}$

dipole $r \gg a$



$$\vec{A}(\vec{r},t) = \frac{\mu_0 q \vec{v}(t)}{4\pi r}\left(1 + \frac{\vec{r}.\vec{w}(t)}{r^2}\right) \qquad (1)$$

If the condition $\dot{\Psi} \gg \dot{\varphi}cos\theta$ is not verified, the dipole moment cannot be defined since:

$$\vec{A}(\vec{r},t) \neq \frac{\mu_0 \vec{M}(t) \times \vec{r}}{4\pi r^3}$$

$<L_1> \neq 0$

Once again, we have : $\vec{L} = \frac{I_1}{M_s^2}\left(\vec{M} \times \frac{d\vec{M}}{dt}\right) + L_3\vec{e}_3$

<u>Goal</u>: derivation of the LLG equation with inertia from formula (1)

Outline

I) Beyond the precession of the magnetization: the inertial regime and nutation.

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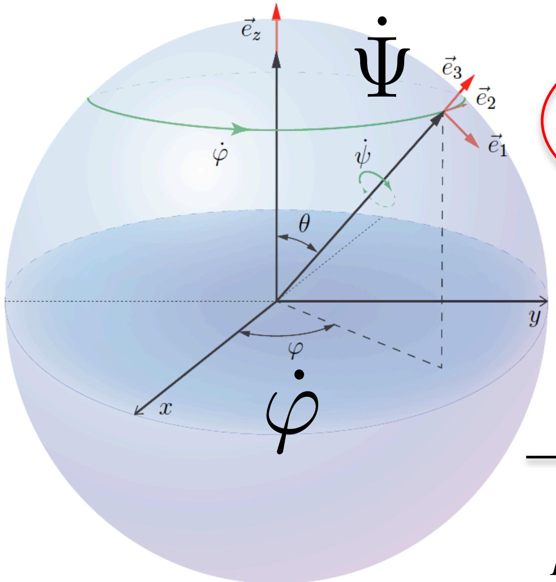
The geometric phase?
A tool for the separation between slow and fast variables

Question 1: What is the number of full rotations of the stick around his axis during the time of one precession?



I) without inertia

Larmor pulsation:

$$\Omega_\varphi = \dot{\varphi} = \frac{M_s H_z}{L_3} = -\gamma H_z$$

Duration of one precession:

$$t_s = \left| \frac{2\pi}{\Omega_\varphi} \right| = \frac{2\pi}{\gamma H_z}$$

$$\Delta\Psi_0 = \int_0^{t_s} \dot{\psi}\, dt = (\Omega_3 - \dot{\varphi} cos\theta) t_s \quad = 2\pi \left(\frac{M_s}{\gamma^2 I_3 H_z} + cos\theta \right)$$

Anticipating the inertial case:
Introduction of the « Slowness parameter G »

$$G = \frac{L_3}{\sqrt{I_1 M_s H_z}} = \frac{1}{\gamma H_z} \sqrt{\frac{M_s H_z}{I_1}} = \frac{1}{2\pi} \frac{t_s}{t_0}$$

$$t_0 = \sqrt{\frac{I_1}{M_s H_z}}$$

Fast characteristic time

Dynamical phase

Geometric phase?
(Hannay angle)

$$\Delta\Psi_0 = 2\pi \left(\frac{I_1}{I_3} G^2 + cos\theta \right)$$

Question 2: what is the physical signification of this phase?

$\Delta\Psi_0$ defines the flux of the magnetic monopole

Assuming a magnetic monopole.

Circulation of $\vec{A}$ along a loop of in the configuration space, defines a flux of an axial vector $\vec{B}$:

$$\Delta\Psi_0 \equiv \oint \vec{A}.d\vec{l} = \int\int \vec{B}.d\vec{S} = 2\pi R^2 B$$

where $\vec{B} = B\,\vec{e}_3$ such that $\vec{B} = rot(\vec{A})$

we have shown that:

$$\Delta\Psi_0 = 2\pi\left(\frac{M_s}{\gamma^2 I_3 H_z} + cos\theta\right)$$

so that:

$$B = -\frac{M_s}{\gamma^2 I_3 H_z R^2} + \frac{cos\theta}{R^2}$$

What does it mean?

Atomicstic parameters
(Bohr radius a)

Bohr magneton

$$M_s = \mu_B = \gamma\hbar$$

Gyromagenic ratio

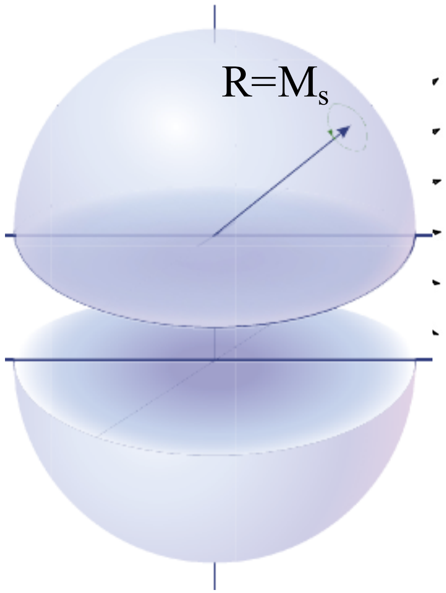
$$\gamma = q/(2m)$$

Moment of inertia

$$I_3 = ma^2$$

Quantum flux due to H_z

$$\Phi_0 = H_z \pi a^2 cos\theta = h/q$$



configuration space

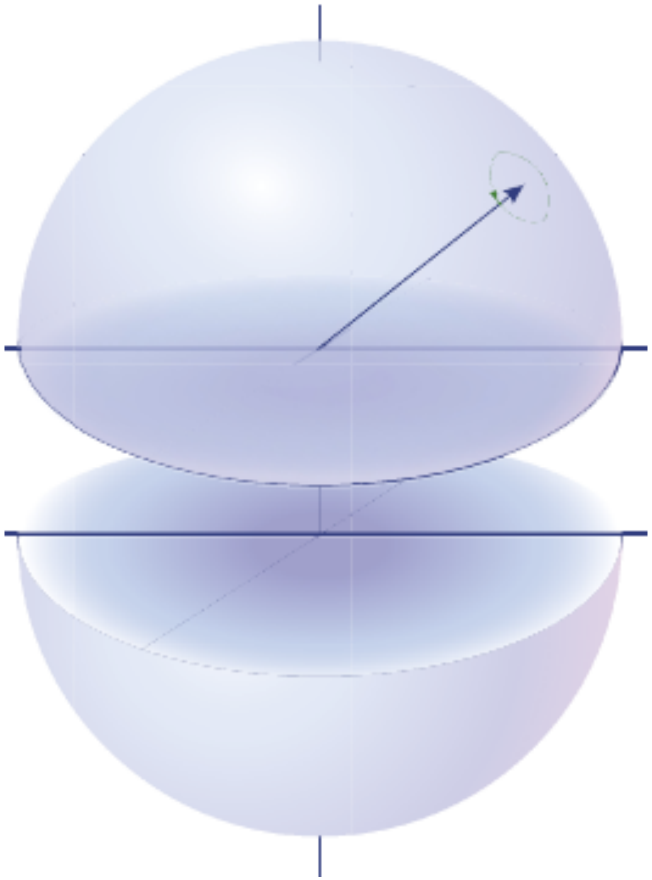
$$\Rightarrow \quad \boxed{B = \frac{4\,cos\theta}{R^2}}$$

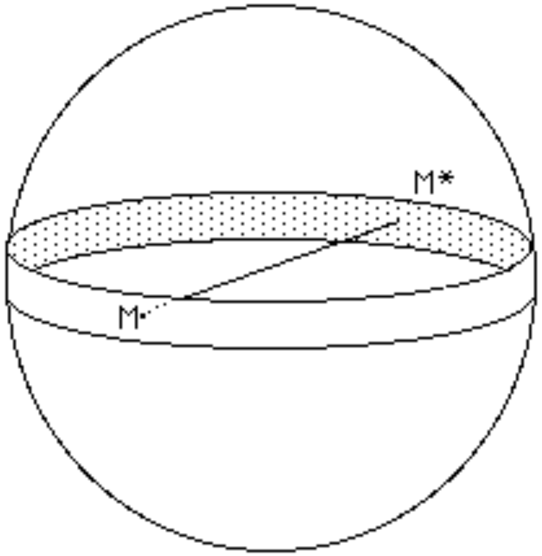
Coincides with Berry field for a magnetic monopole of spin ½ and θ = π

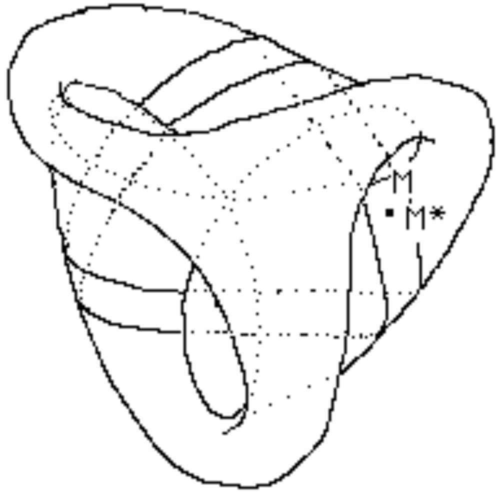
$$\vec{B}_n = -Im \sum_{n' \neq n} \frac{\langle n|\vec{\nabla}\hat{H}|n'\rangle \times \langle n'|\vec{\nabla}\hat{H}|n\rangle}{(E'_n - E_n)^2}$$

Cutting the configuration space in two hemispheres induces strong topological constraints

Equatorial plane= one side surface embedded in R^3 .
Projective plane!







Möbius band

Boy surface

Conclusion (trivial): the dipole is « topologically protected »

Conclusion

- Inertia of the magnetizatyion = Beyond Gilbert's hypothesis:

$$\dot{\Psi} \gg \dot{\varphi} cos\theta$$

the magnetic momentum as inertial magnetic degrees of freedom.

- First experimental evidence. The door is open for further investigation in the context of ultrafast magnetism.

- The electrodynamic model: more than a pedagogical approach.

- Magnetic dipole = topologically protected : do you agree?