P.C.W. Holdsworth
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Emergent Symmetry and Quasi-Particules

1. A review of electrostatics.
2. Emergent charge in 2D and the KT transition
3. Emergent magnetic monopoles in spin ice

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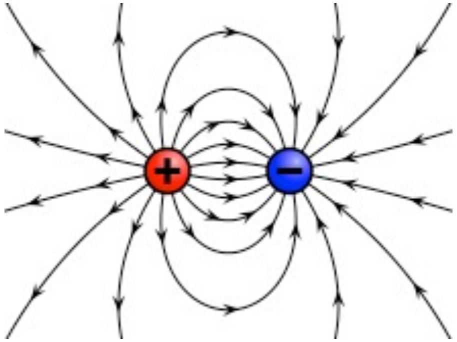
Emergent Symmetry and Quasi-Particules

Elsa Lhotel,
Tom Fennell,
Ludovic Jaubert,
Steven Bramwell,
Roderich Moessner,
Michel Gingras
Andrea Taroni

Flavien Museur
Geoffroy Haeseler
Andres Huster Zapke

Adam Harman-Clarke,
Vojtech Kaiser,
Michael Faulkner,
Valentin Raban,
Marion Brooks-Bartlett,
Callum Gray,
Alexandra Turini

An electrostatic problem

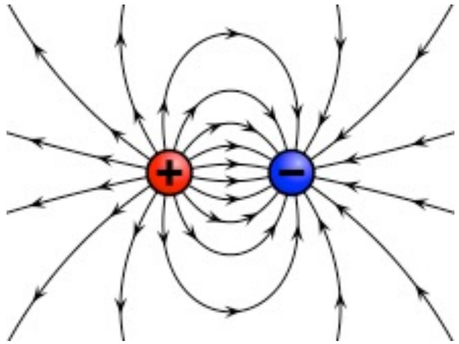


$$\vec{\nabla}.\vec{E}(\vec{r}) = \frac{1}{\varepsilon_0}\rho(\vec{r})$$

Charges $$u_c(r) = \frac{q_1 q_2}{4\pi\varepsilon_0 |\vec{r}|}$$

Fields $$U = \frac{1}{2}\varepsilon_0 \int \vec{E}(\vec{r})^2 d^3\vec{r}$$

An electrostatic problem



$$\vec{\nabla}.\vec{E}(\vec{r}) = \frac{1}{\varepsilon_0}\rho(\vec{r})$$

Charges

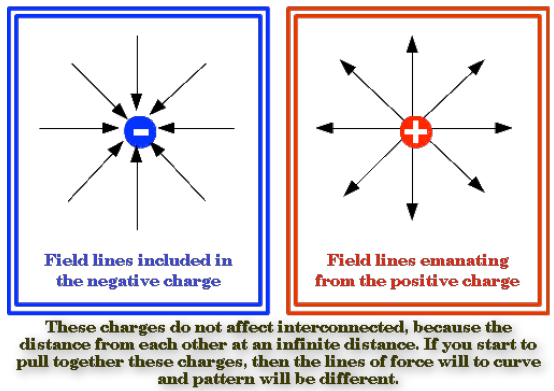
$$u_c(r) = \frac{q_1 q_2}{4\pi\varepsilon_0 |\vec{r}|}$$

Fields

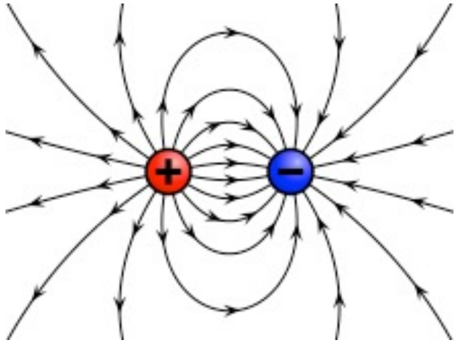
$$U = \frac{1}{2}\varepsilon_0 \int \vec{E}(\vec{r})^2 d^3\vec{r}$$

$$U_{self} = \frac{N}{2}\varepsilon_0 \int \vec{E}(\vec{r})^2_{self} d^3\vec{r}$$

$$u_c = U - U_{self}$$



Field representation and Helmholtz decomposition



Electrostatics:

Gauss' Law $\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$ $\vec{E} = -\vec{\nabla}\phi$ => Poisson $\nabla^2\phi + \frac{\rho}{\varepsilon_0} = 0$

One solution from an infinity of possibilities

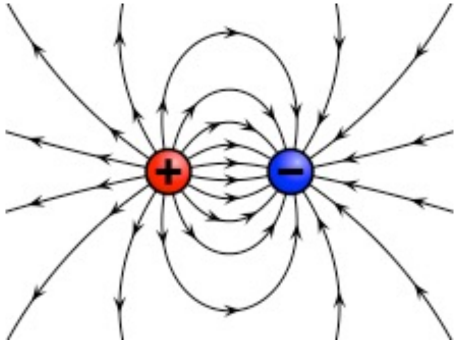
$$\vec{E} = -\vec{\nabla}\phi + \vec{\nabla} \wedge \overrightarrow{A'} = \vec{E}_m + \vec{E}_d$$

$$U = \frac{\varepsilon_0}{2} \int \left(\vec{E}_m{}^2 + \vec{E}_d{}^2\right) d\vec{r}$$

Electrostatics – minimum energy => $\vec{E}_d = 0$

(Classical Electrodynamics – Coulomb gauge => $\vec{E}_d = -\frac{\partial \vec{A}}{\partial t}$, $\vec{p} \rightarrow \vec{p} - q\vec{A}$**)**

Field representation and Helmholtz decomposition



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One solution from an infinity of possibilities

$$\vec{E} = -\vec{\nabla}\phi + \vec{\nabla} \wedge \overrightarrow{A'} + \vec{h} = \vec{E}_m + \vec{E}_d + \vec{E}_h$$

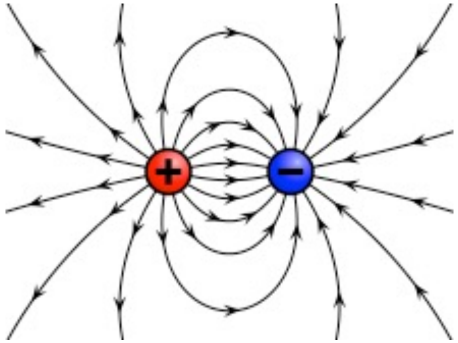
$$U = \frac{\varepsilon_0}{2} \int \left(\vec{E}_m^2 + \vec{E}_d^2 + \vec{E}_h^2\right) d\vec{r}$$

Constant "harmonic" term

Electrostatics – minimum energy => $\vec{E}_d = 0$

(Classical Electrodynamics – Coulomb gauge => $\vec{E}_d = -\frac{\partial \vec{A}}{\partial t}$, $\vec{p} \rightarrow \vec{p} - q\vec{A}$)

Field representation and Helmholtz decomposition



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Emergent examples in condensed matter – Ex. XY magnets and spin ice =>

$\vec{E}_m \neq 0,$ $\vec{E}_d \neq 0$ **NO classical (electro)dynamics => mass $m = \infty$**

Quantum fluctuations CAN give emergent quantum electrodynamics

Harmonic field and topological invariants

Poisson's equation on a torus is invarient under the addition of a constant winding field

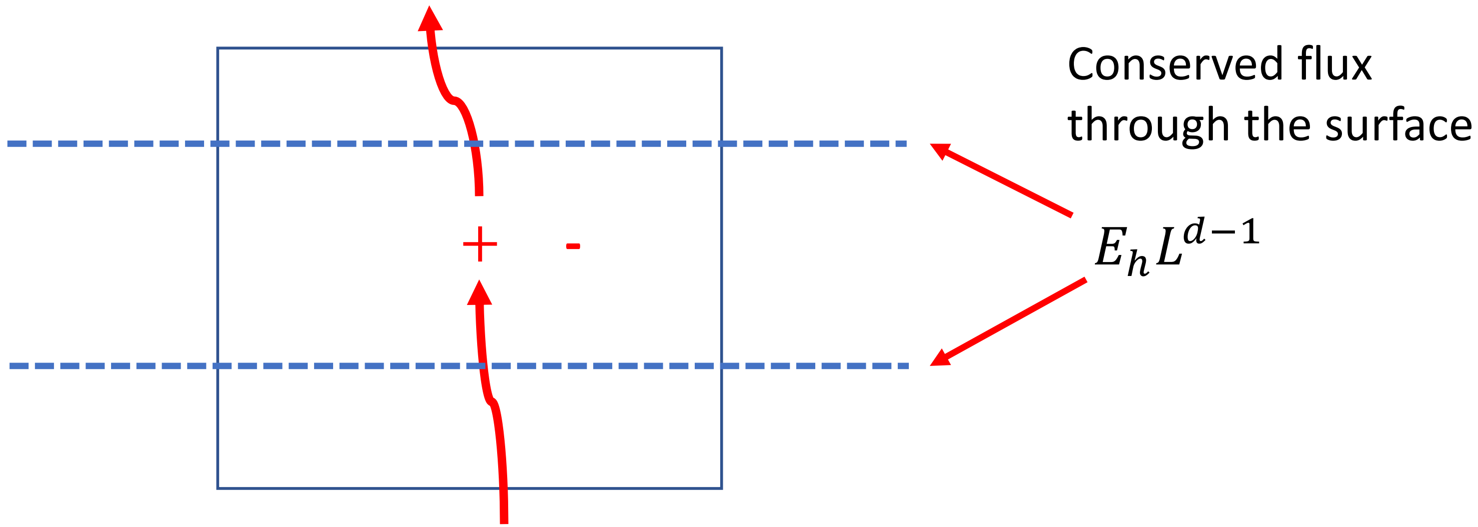
$$\nabla^2 \phi = \frac{1}{\varepsilon_0}\rho(\vec{r}),\ \phi \Longrightarrow \phi' = \phi + \vec{E}_h . \vec{r}$$

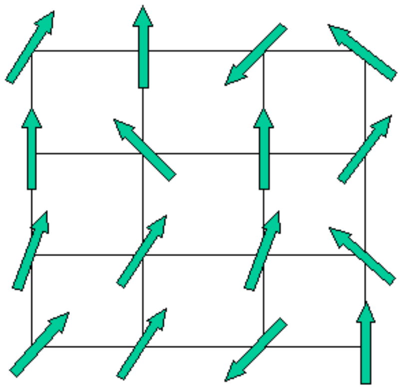
$$\vec{E}_h = E_h \hat{z} \qquad \delta E_h \sim \frac{q}{L^{d-1}}$$

$\boldsymbol{\vec{E}_h = \omega \delta E_h \hat{z}}$

ω is an integer

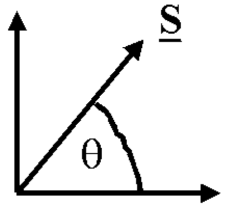
Constant everywhere in the box





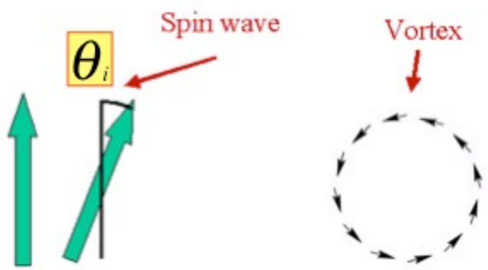
2D-XY model

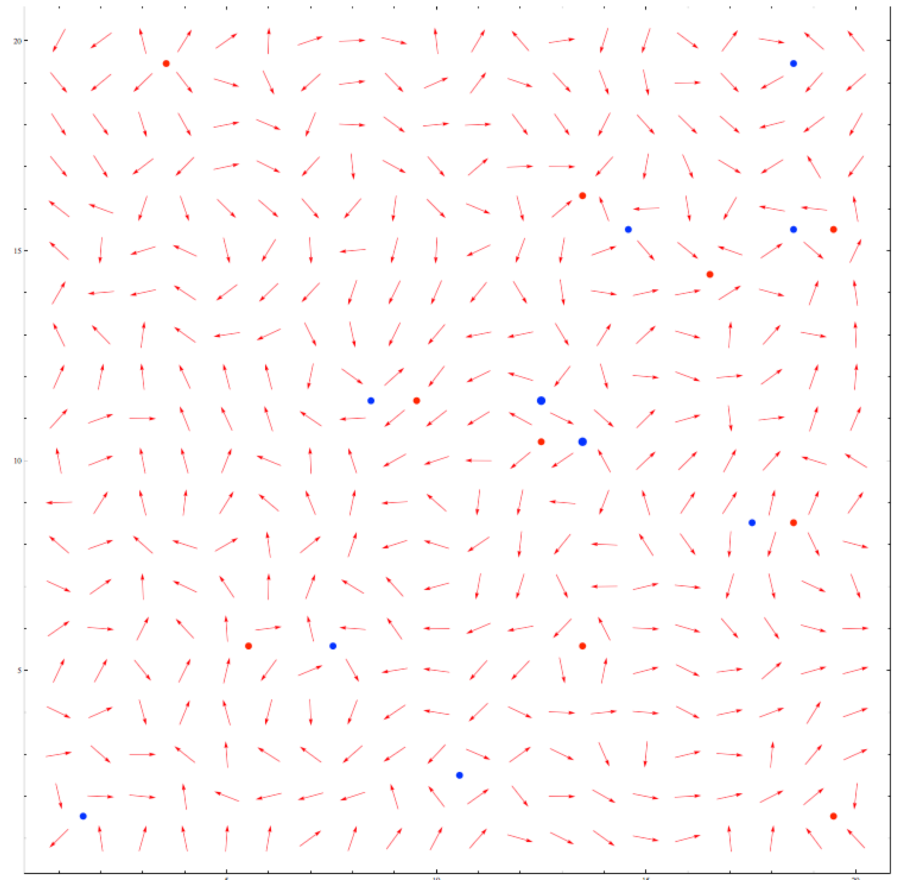
$$H = -J \sum_{<i,j>} \cos(\theta_i - \theta_j)$$



No long range order at any Temperature Mermin Wagner 1966

Two classes of excitation



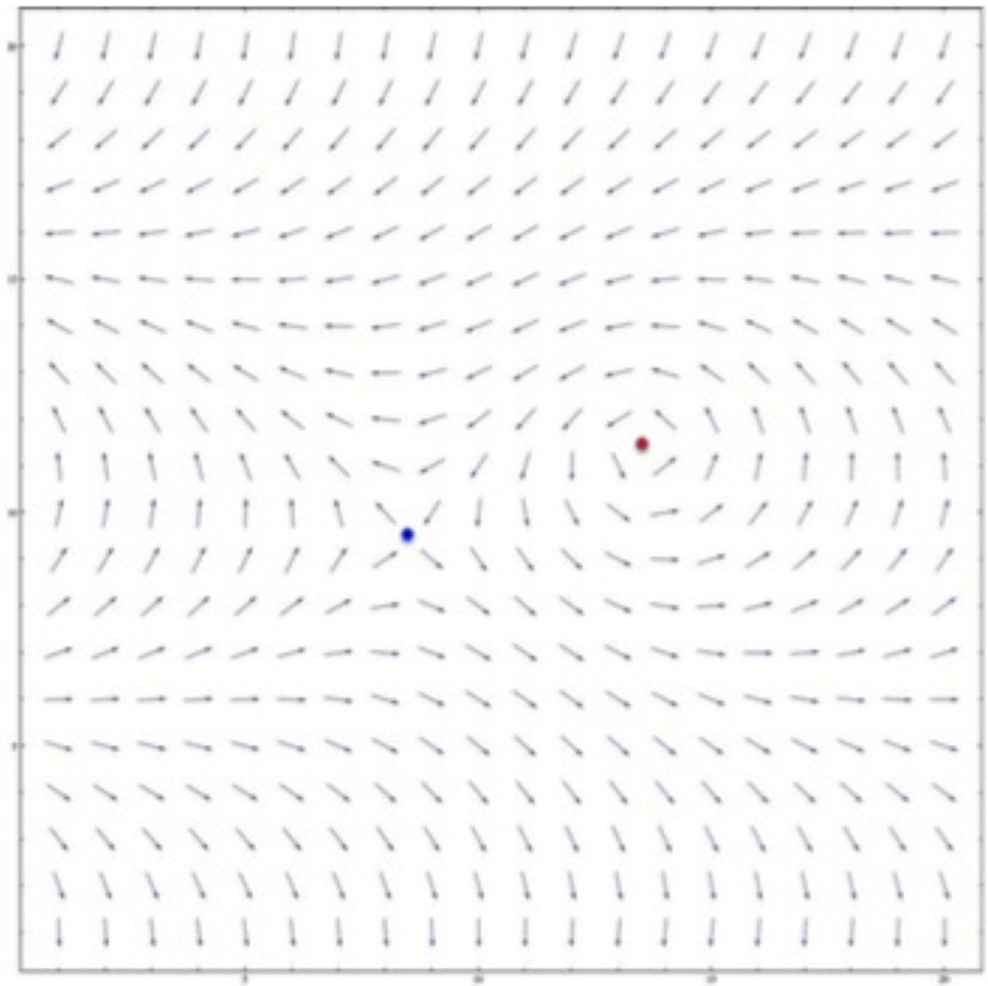


But there is a phase transition!

V. L. Berezinski , Sov. Phys.—JETP 32 493 (1970)
J M Kosterlitz and D J Thouless 1973 J. Phys. C: Solid State Phys. 6 1181

(dielectric-electrolyte transition –
Salzberg and Prager J. Chem. Phys. 38, 2587, 1963)

Freezing spin wave
Excitations shows vortices
With long range interactions



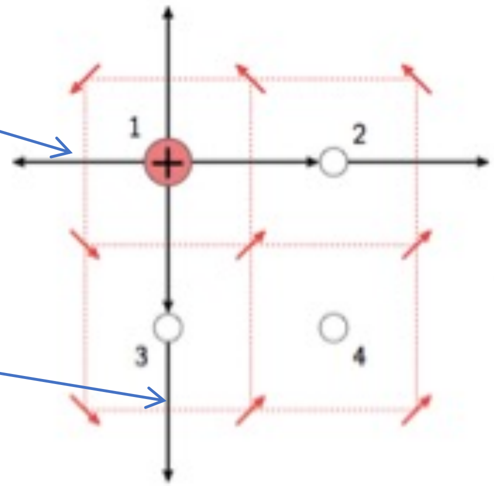
Kosterlitz-Thouless phase
Transition involves the
Creation and unbinding of
Vortex pairs

M. Faulkner, Thesis ENS-UCL 2014

Spin field maps to an "Electric field" on duel bonds

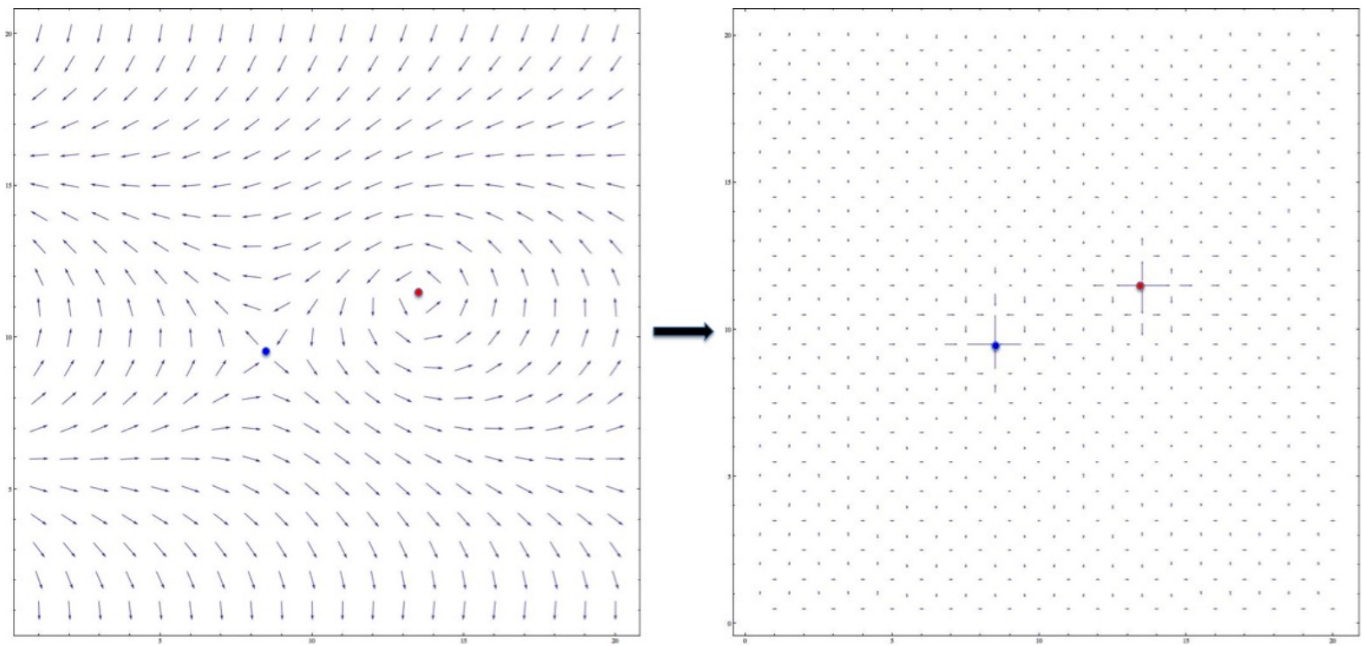
$$E_x = \partial_y \theta,$$

$$E_y = \partial_x \theta,$$



Spins

Field



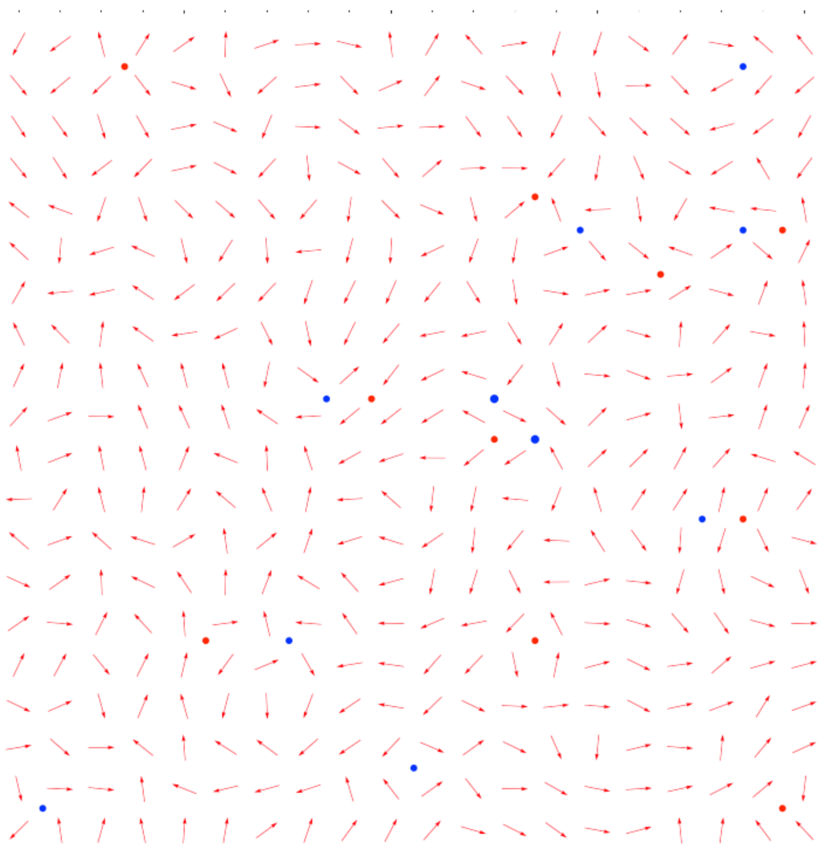
Spin field => emergent E

$\theta(\vec{r}) \to \vec{E}(\vec{r})$

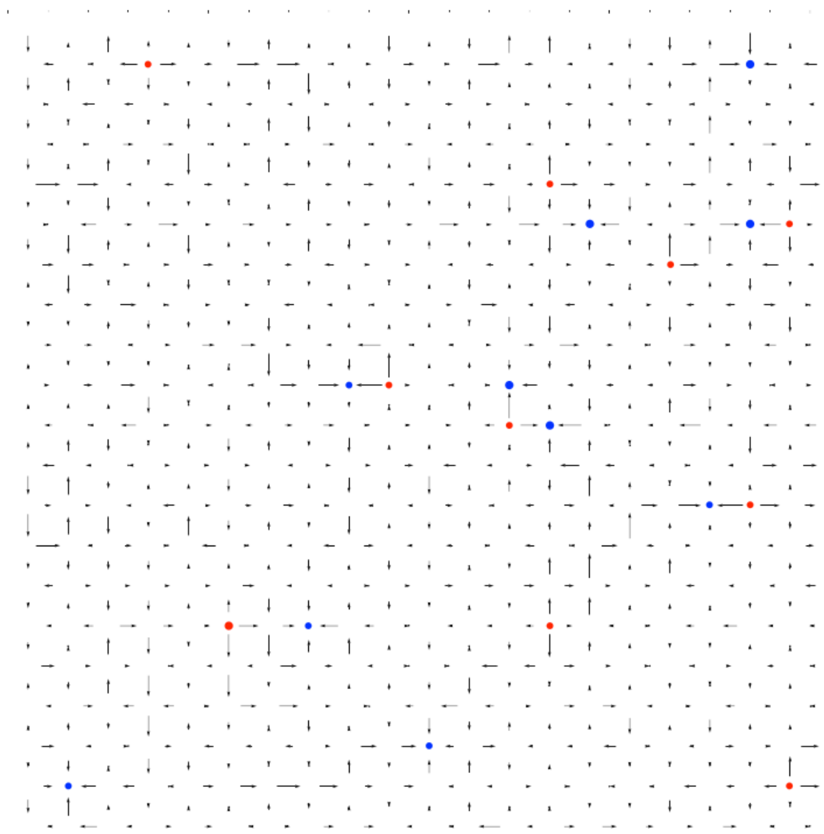
$$\vec{E} = -\vec{\nabla}\phi + \vec{\nabla} \wedge \overrightarrow{A'} = \vec{E}_m + \vec{E}_d$$

Vortex part + spin wave part

2D equivalent of the generalized electrostatic problem



=>



An exact mapping is found in the Villain model

J. Villain, J. Phys (France), 36, 581, (1975)

Coulomb interaction in 3D is NON-confining

$$U(r) = \frac{\mu_0}{4\pi} \frac{Q_i Q_j}{r};$$ $$U(r = a) = constant$$ $$U(r \to \infty) = 0$$

Bound pairs will separate

Coulomb interaction in 2D is confining

$$U(r) \sim \log(r)$$ $$S(r) \sim \log(r)$$

Confinement deconfinement transition at T_{KT}

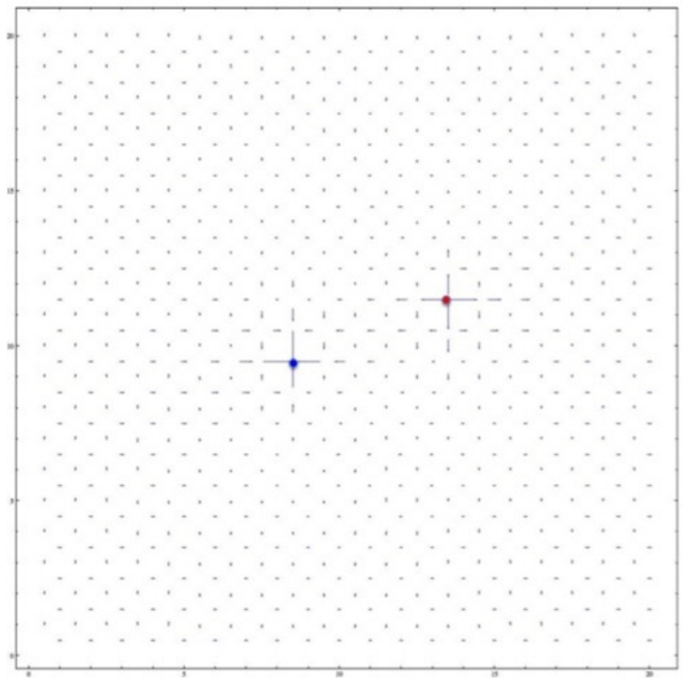
The Nobel Prize in Physics 2016

© Trinity Hall, Cambridge University. Photo: Kiloran Howard
David J. Thouless
Prize share: 1/2

Photo: Princeton University, Comms. Office, D. Applewhite
F. Duncan M. Haldane
Prize share: 1/4

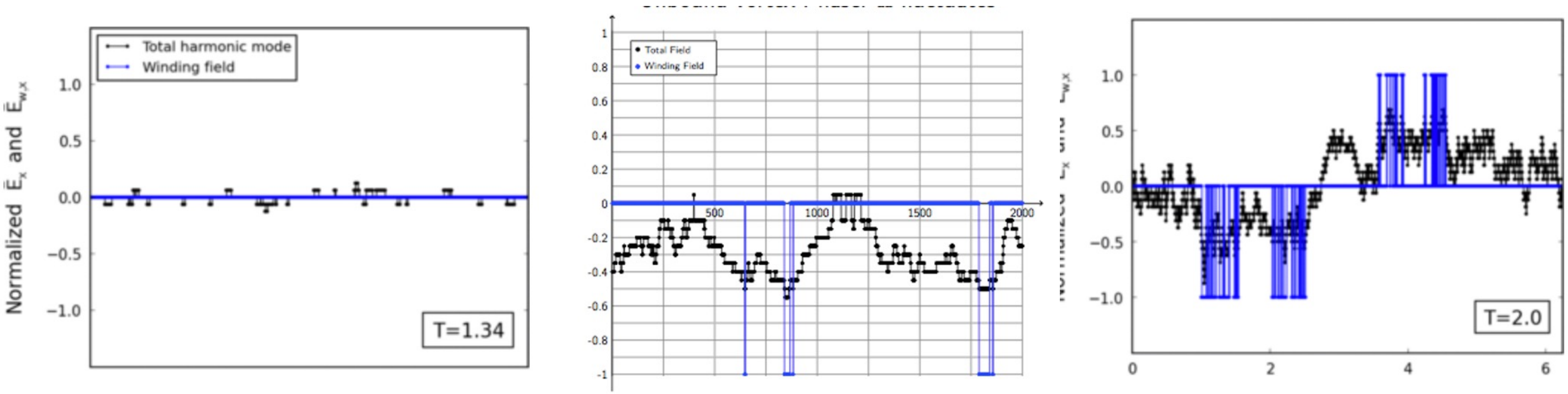
Ill: N. Elmehed. © Nobel Media 2016
J. Michael Kosterlitz
Prize share: 1/4

The Nobel Prize in Physics 2016 was divided, one half awarded to David J. Thouless, the other half jointly to F. Duncan M. Haldane and J. Michael Kosterlitz *"for theoretical discoveries of topological phase transitions and topological phases of matter"*.



Topological Invariant Fluctuations through KT-transition

$T < T_{KT}$ $T \sim T_{KT}$ $T > T_{KT}$

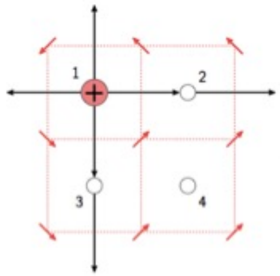


Topological contribution to harmonic field

Total – harmonic plus Coulomb gas contributions

Michael F. Faulkner, Steven T. Bramwell, and Peter C. W. Holdsworth, Phys. Rev. B 91, 155412, (2015)

Experimental realisations:

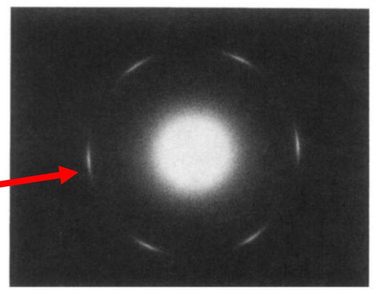


As vortices are 4-point objects they are notoriously difficult to see with 2-point probes

Indirect measures – power law correlations

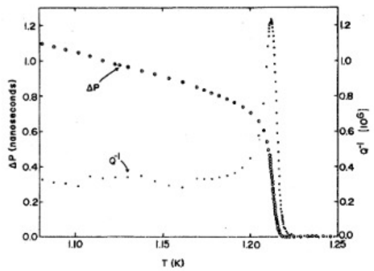
Ex – hexatic liquid crystal films

Broad peaks indicate Power law correlations


(b)

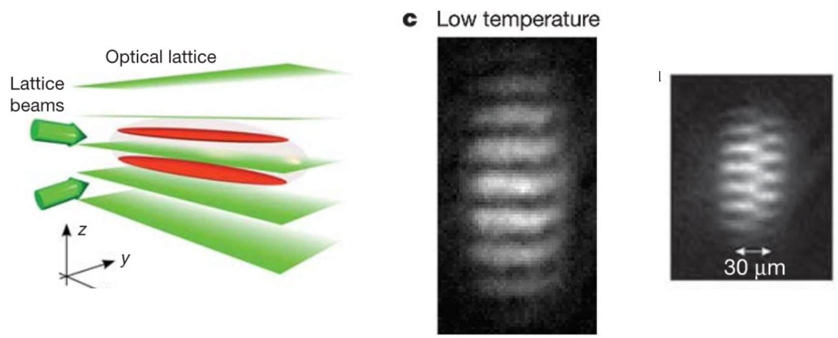
Ming Cheng, John T. Ho, S. W. Hui, and Ronald Pindak Phys. Rev. Lett. 61, 550 (1988)

“Universal jump” in the helicity modulus
Torsion balance experiments in ^{4}He thin film
Bishop and Reppy, Phys. Rev. B 22, 5171, 1980



Cold atoms – phase interference between 2D fluids

Zoran Hadzibabic, Peter Krüger, Marc Cheneau, Baptiste Battelier & Jean Dalibard
Nature volume 441, pages 1118–1121 (2006)

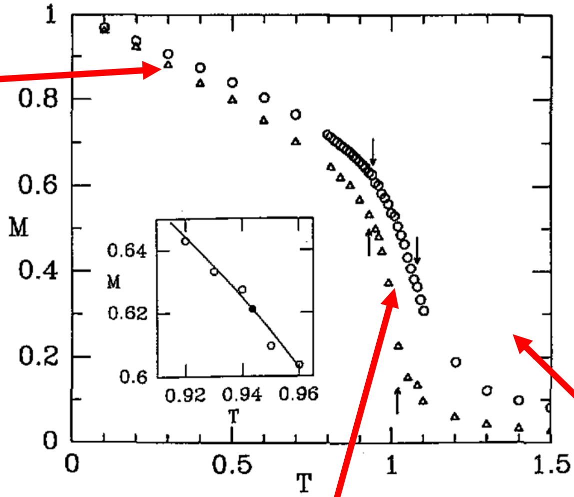


Finite size magnetisation – Bramwell, Holdsworth J.Phys:. Condens. Matter 5 (1993) L53459

$M(N \longrightarrow \infty) = 0$ **BUT – non-linear change in finite size M at T_{KT}**

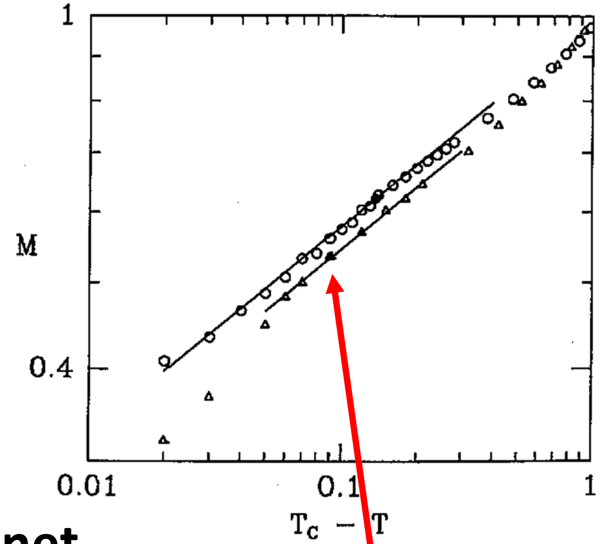
N=1000, N=10000

QLRO



Paramagnet

Universal with $\beta_{eff} \sim 0.23$



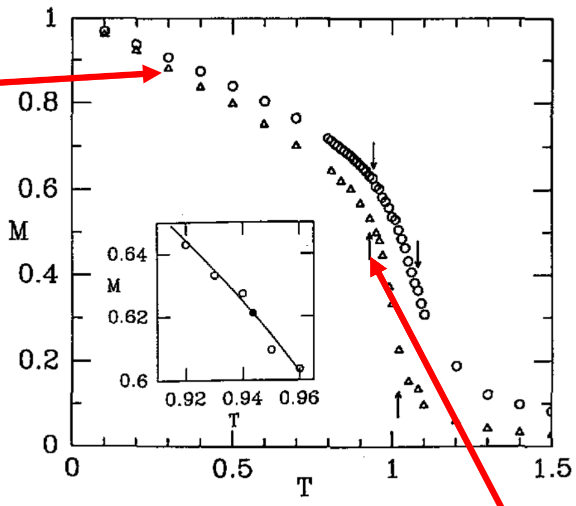
$M = M_0(T_c - T)^{0.23}$

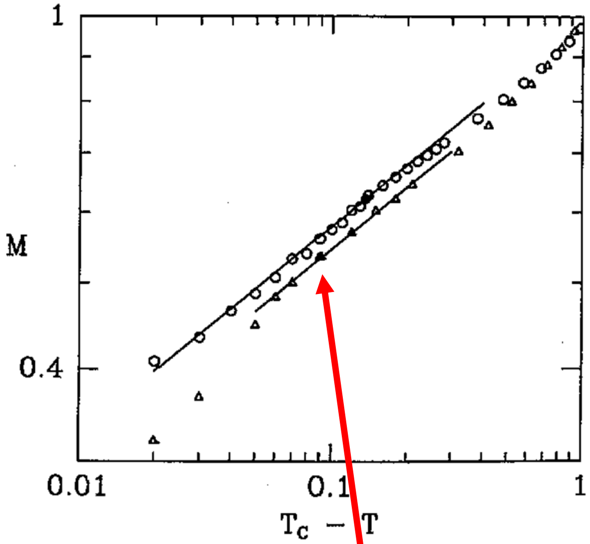
Finite size magnetisation – Bramwell, Holdsworth J.Phys:. Condens. Matter 5 (1993) L53459

$M(N \longrightarrow \infty) = 0$ **BUT – non-linear change in finite size M at T_{KT}**

N=1000, N=10000

QLRO





$$\frac{\partial \ln(M)}{\partial ln(T_c - T)} = \frac{3\pi^2}{128} = 0.2313 \ldots$$

$$M = M_0(T_c - T)^{0.23}$$

Manipulating Kosterliz' RG equations we find

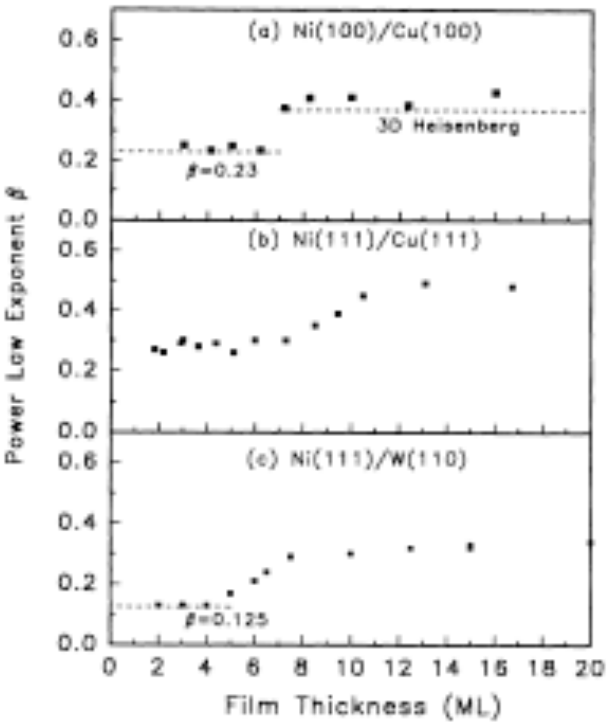
Kosterlitz J M 1974, J. Phys. C: Solid State Phys. 7 1046

Finite size magnetisation – Bramwell, Holdsworth J.Phys:. Condens. Matter 5 (1993) L53459

$M(N \longrightarrow \infty) = 0$ **BUT – non-linear change in finite size M at T_{KT}**

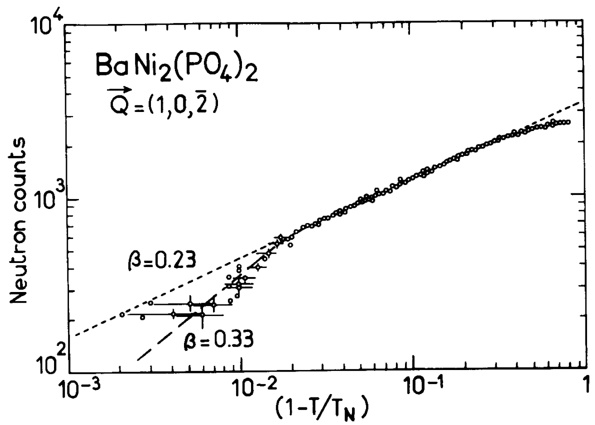
Ni films on Cu(100), Cu(111), W(110)

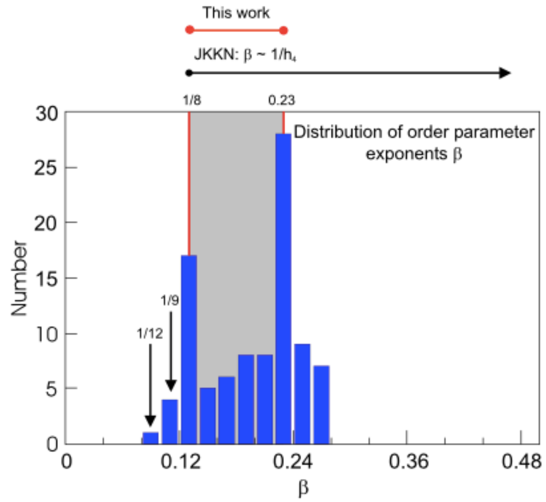
Huang et al, PRB 49 3962 (1994)



Crossover in quasi-2D magnets

Regnault L P and Rossat-Mignod J 1990 Magnetic Properties of Layered Transition Metal Compoundsed L J de Jongh (Kluwer–Academic)

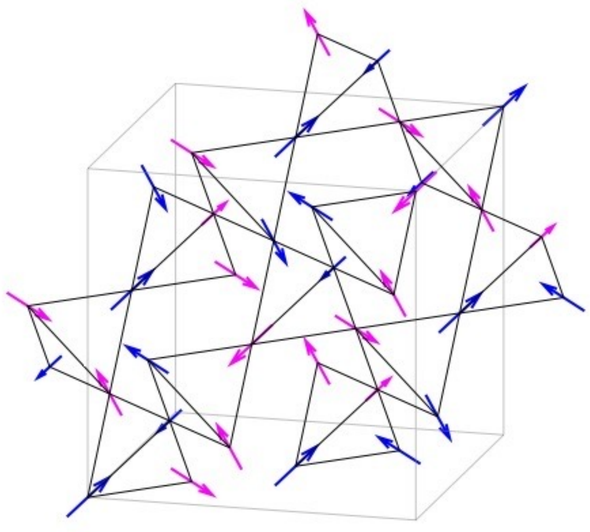




Taroni, Bramwell, Holdsworth,
J. Phys.: Condens. Matter 20 (2008) 275233

Spin Ice Materials

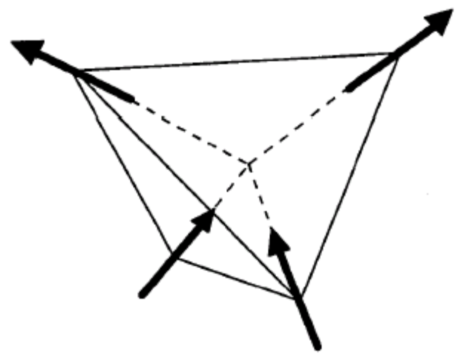
Spin ice materials $Ho_2Ti_2O_7$, $Dy_2Ti_20_7$ (HTO,DTO)



Harris et al, Phys. Rev. Lett.**79**, 2554-2557 (1997)

Discrete orientation - in or out

Lowest energy => Magnetic ice rules two-in two-out



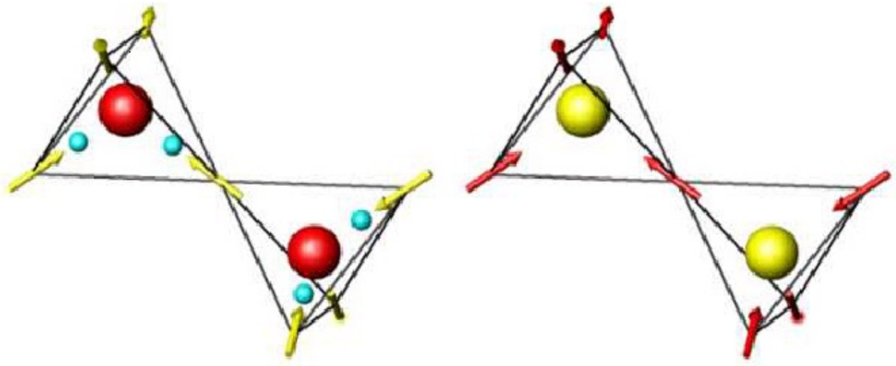
Pyrochlore lattice= corner sharing tetrehadra whose centres form a diamond lattice

Harris et al, Phys. Rev. Lett. **79**, 2554-2557 (1997)

Spin Ice Materials

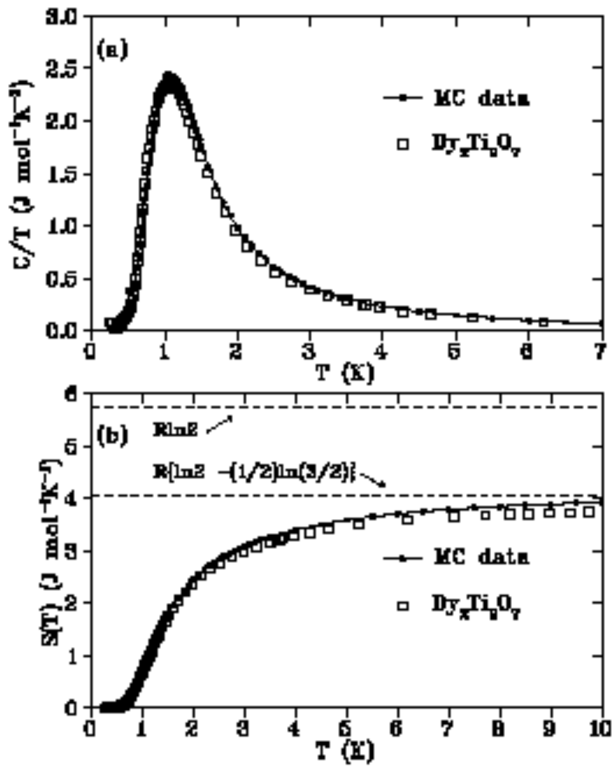
$Ho_2Ti_2O_7$, $Dy_2Ti_2O_7$

Magnetic ice rules => Pauling entropy.



$$S_P \approx N k_B \frac{1}{2} \ln\left(\frac{3}{2}\right)$$

Magnetic « Giauque and Stout » experiment:



Ramirez et al, Nature 399,333, (1999)
D. Pomaranski et al., Nat. Phys.
DOI: 10.1038/NPHYS2591, 2013 ,
Giblin et al., Phys. Rev. Lett. 121, 067202 (2018)

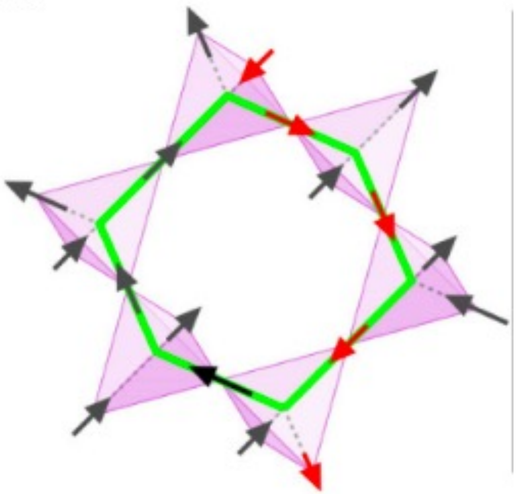
No Phase Transition in zero field !

Spin Ice Models

Spin ice materials $Ho_2Ti_2O_7$, $Dy_2Ti_20_7$

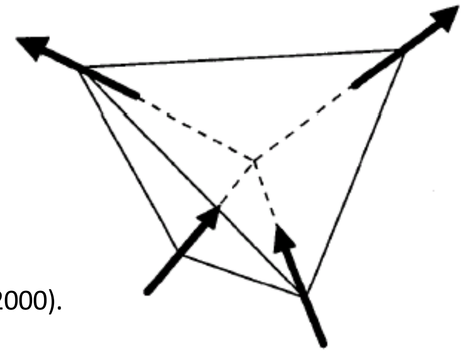
Magnetic ice rules
two-in two-out=>
Topological constraint

(a)



Dipolar Spin Ice Model (DSIM)

B. C. den Hertog and M. J. Gingras, Phys. Rev. Lett., 84, 3430 (2000).



$$H = J\sum_{i,j} \vec{S}_i.\vec{S}_j - Dm^2\sum_{i,j}\left[\frac{\vec{S}_i.\vec{S}_j}{|\vec{r}_{ij}|^3} - \frac{3(\vec{S}_i.\vec{r}_{ij})(\vec{S}_j.\vec{r}_{ij})}{|\vec{r}_{ij}|^5}\right] \sim -J_{eff}\sum \vec{S}_i.\vec{S}_j$$

Long range interaction almost screened in 2 in – 2 out state

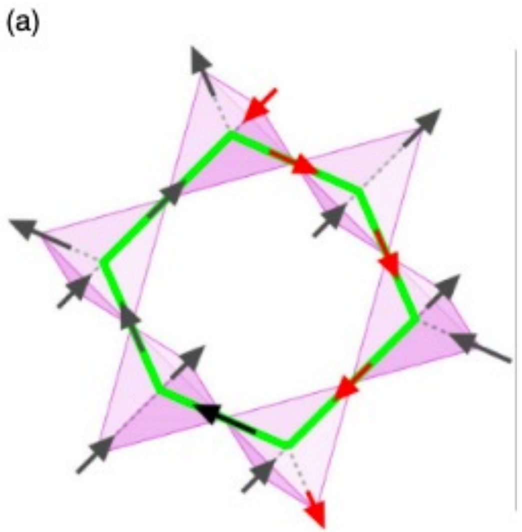
NN Model

Low energy entropy S_p

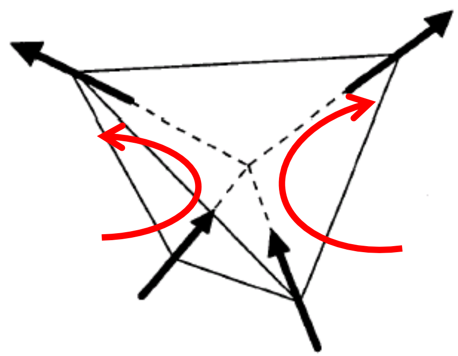
Spin Ice Models

Spin ice materials $Ho_2Ti_2O_7$, $Dy_2Ti_20_7$

Magnetic ice rules
two-in two-out=>
Topological constraint



$$\vec{\nabla}.\vec{M} = 0$$



Spin = Element of a lattice field => ground state condition

Dumbbell model: → => S ●——● N Flux= $\frac{m}{a}$

Point dipole

a

Dumbbell

Möller and Moessner PRL. 96, 237202, 2006, Castelnovo, Moessner, Sondhi, Nature, 451, 42, 2008

Spin Ice Models

Spin ice materials $Ho_2Ti_2O_7$, $Dy_2Ti_20_7$

Magnetic ice rules
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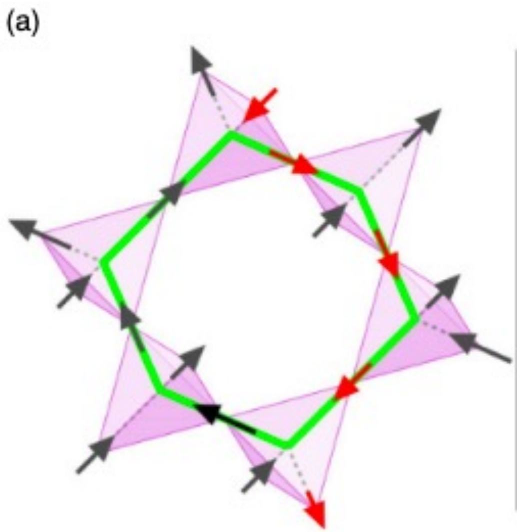
Topological constraint



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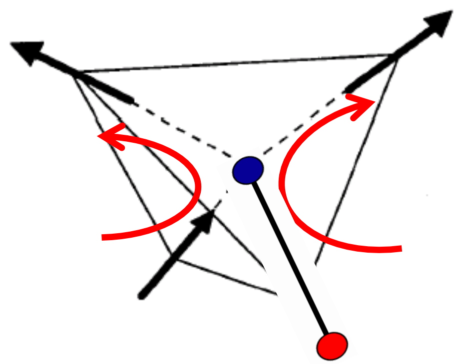

Spin = Element of a lattice field => ground state condition

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Point dipole

a

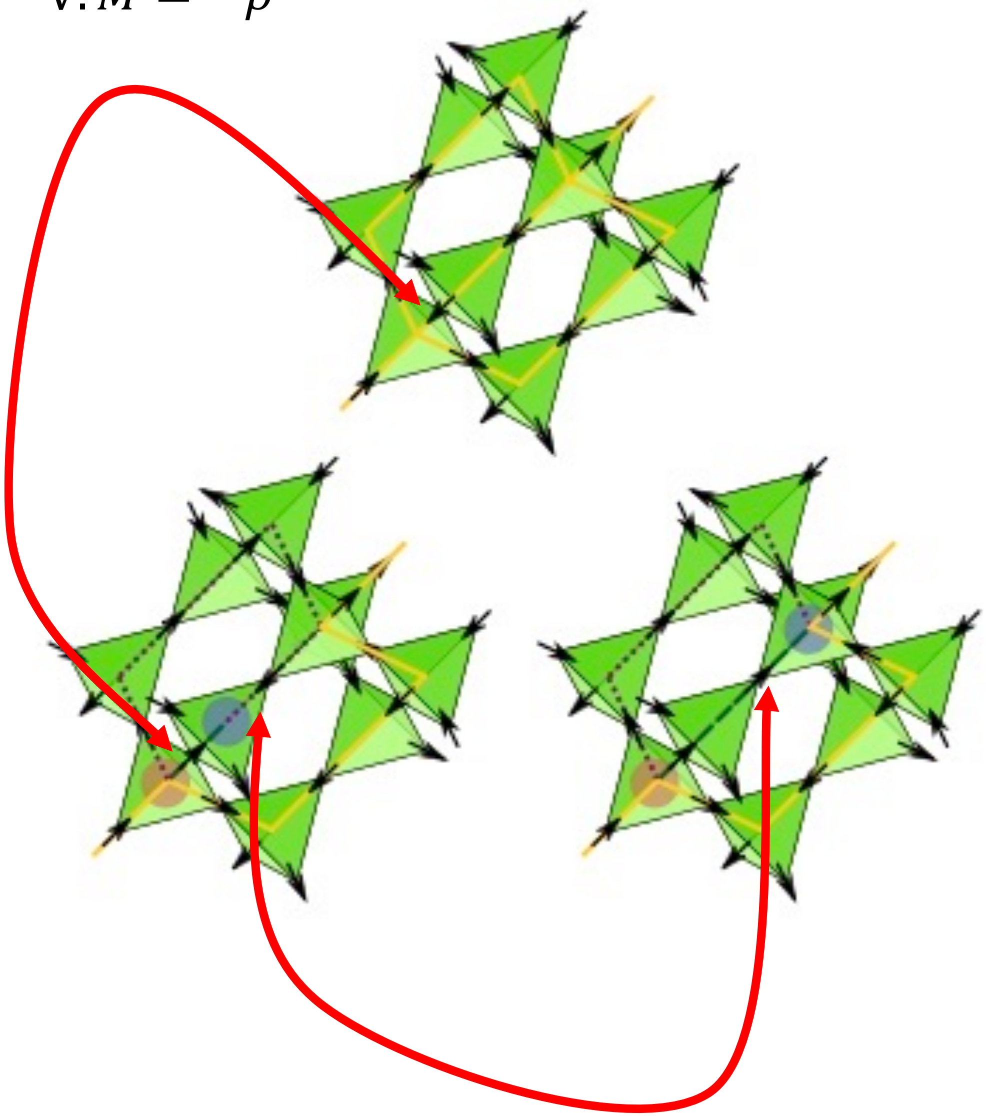
Dumbbell

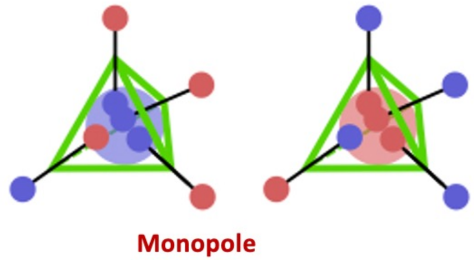
Möller and Moessner PRL. 96, 237202, 2006, Castelnovo, Moessner, Sondhi, Nature, 451, 42, 2008

Excitations are topological defects that break the constraint.

Castelnovo, Moessner, Sondhi, Nature, 451, 42, 2008, Ryzhkin JETP, **101**, 481, 2005, Jaubert and Holdsworth, Nature Physics 5, 258 - 261 (2009).

$$\vec{\nabla}.\vec{M} = -\rho$$





3 out- 1 in

3 in 1 out

Monopole charge $Q = \frac{2m}{a}$

Effective Coulomb int.
Between defects

$$U(r) = \frac{\mu_0}{4\pi} \frac{Q_i Q_j}{r};$$

Helmholtz Decomposition of the magnetization field

Fragmentation

Brooks-Bartlett et al, PRX 4, 011007, 2014
E. Lhotel et al J. LTP 201, 710 (2020)

$$\boldsymbol{M} = \vec{\nabla}\psi + \vec{\nabla}\wedge\vec{A} + \vec{h} = \boldsymbol{M}_m + \boldsymbol{M}_d + \boldsymbol{M}_h$$

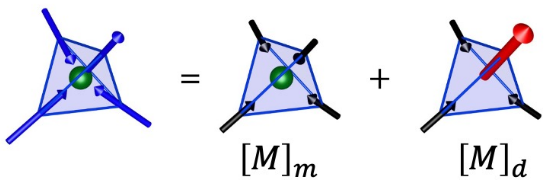
Any monopole configuration can be fragmented. Each element decomposes

$$\boldsymbol{M}_{IJ} = \boldsymbol{M}_{IJ}^{m} + \boldsymbol{M}_{IJ}^{d} + \boldsymbol{M}_{IJ}^{h}$$

Real space iterative method developed by Slobinsky, Pili, and Borzi, Phys. Rev. B 100, 020405(R) (2019).

Emergent continuous symmetry from Ising degrees of freedom

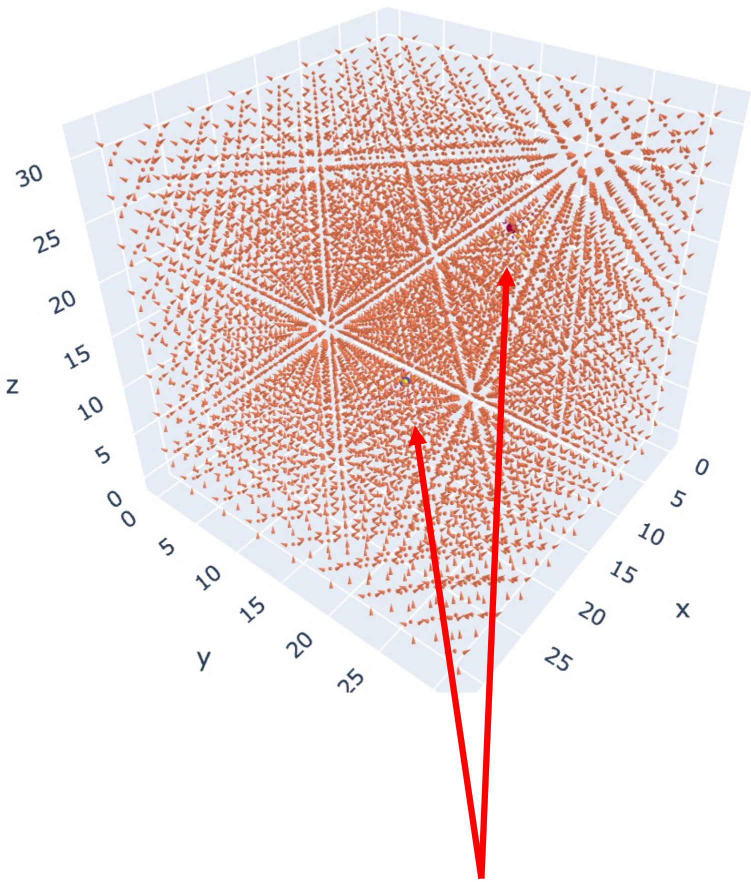
Ex: Single isolated monopole – a 3in – 1out vertex



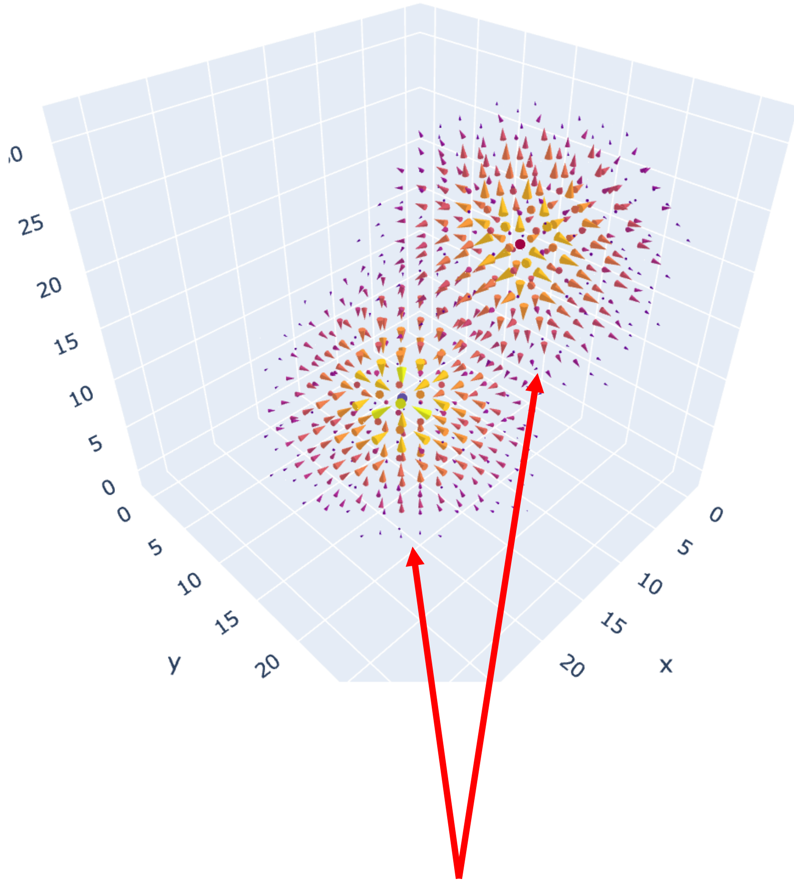
$[M]_h = 0$

$$\left[M_{ij}\right] = \frac{m}{a}(-1,-1,-1,1) = \frac{m}{a}\left(-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2}\right) + \frac{m}{a}\left(-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},\frac{3}{2}\right)$$

Decomposition for a monopole pair



Spin config and monoopole pair



$[M_{IJ}^{m}]$

Consequences:

1. Internal energy is longitudinal only =>$U = U_m \sim \sum_{IJ} \left(M_{IJ}^m\right)^2$

2. Interaction with external field is harmonic only => $U_B = -[\vec{M}_{IJ}^h].\vec{B}$

3. Any leftover part

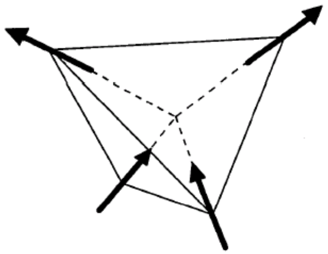
$$\boldsymbol{M_{IJ}^d} = \boldsymbol{M_{IJ}} - \boldsymbol{M_{IJ}^m} - \boldsymbol{M_{IJ}^h}$$

Contributes zero (classical) energy and retains the entropy of loops

4. Partially ordered phases (monopole crystal, spin ice in [111] field)

5. Quantum fluctuations are restricted to $\boldsymbol{M_{IJ}^d}$

Neutron scattering from monople vacuum



$$\vec{\nabla}.\vec{M}=0$$

Pinch Points:

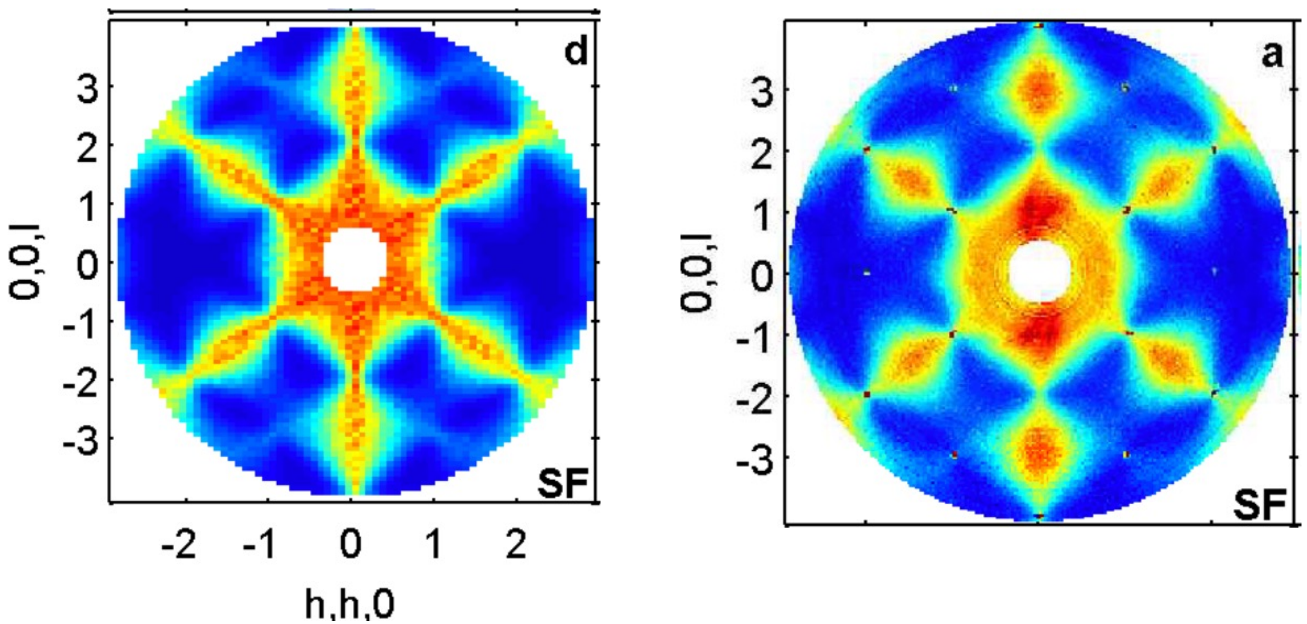
$$\vec{q}.\vec{M}(\vec{q})=0$$

$$\vec{M}=\vec{\nabla}\wedge\vec{A}=\vec{M}_d$$

Monopole vacuum - emergent « Coulomb phase » Physics.

Henley doi.org/10.1146/annurev-conmatphys-070909-104138

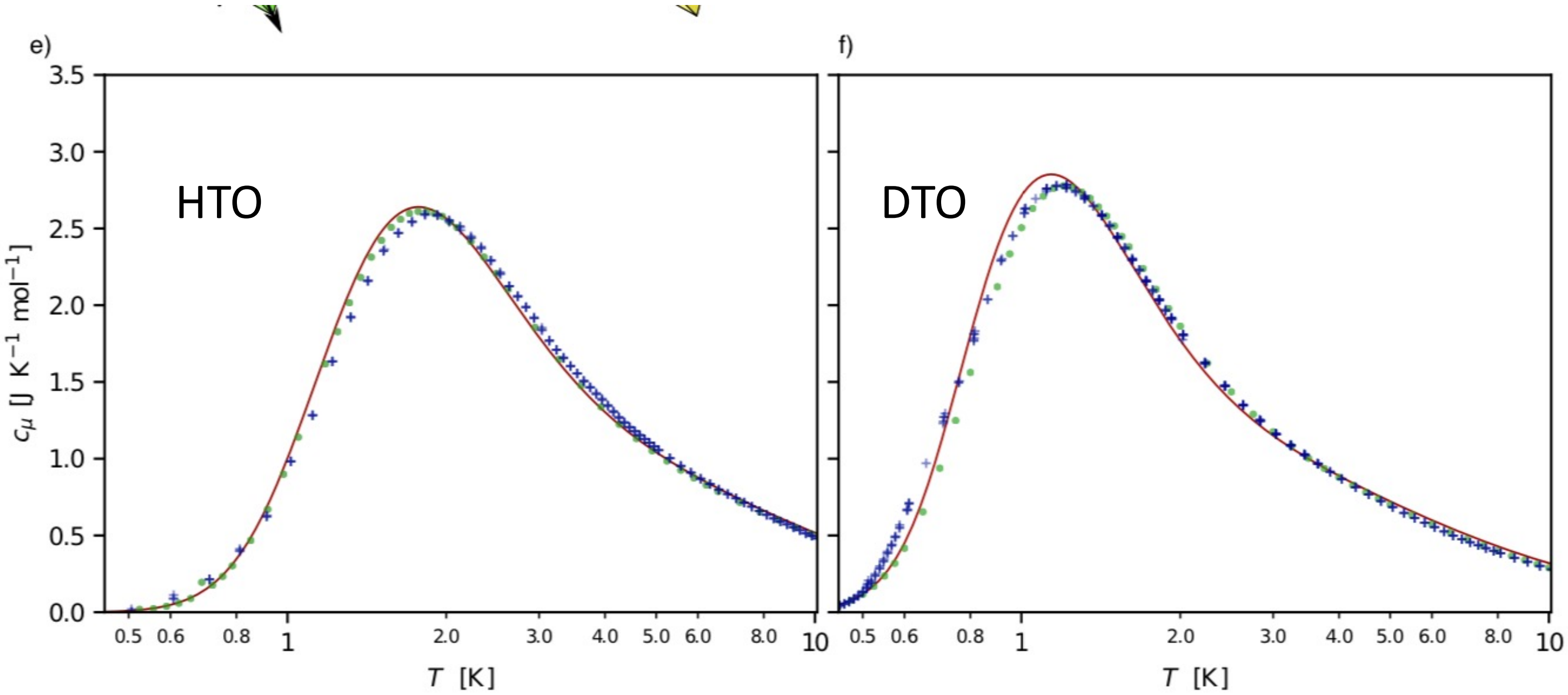
S. V. Isakov, K. Gregor, R. Moessner, and S. L. Sondhi PRL 93, 167204, 2004



T. Fennell *et. al.*, Magnetic Coulomb Phase in the Spin Ice $Ho_7O_2Ti_2$ *Science*, 326, 415, 2009.

Energy fluctuations from quasi-particle picture

Specific heat of spin ice as a Coulomb fluid (magnetolyte)



Vojtech Kaiser, Jonathan Bloxsom, Laura Bovo, Steven T. Bramwell, Peter C.W. Holdsworth, Roderich Moessner ,
Phys. Rev. B 98, 144413 , 2018

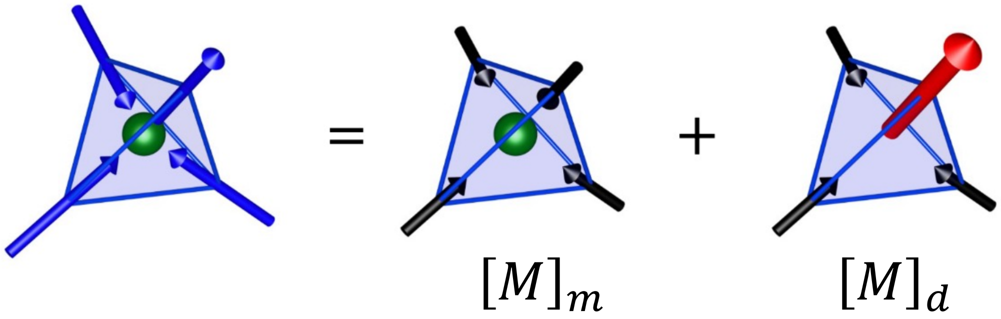
- Experiment
- Simulation- dumbbell model (magnetolyte)
- Debye-Huckel-Bjerrum theory – our best shot

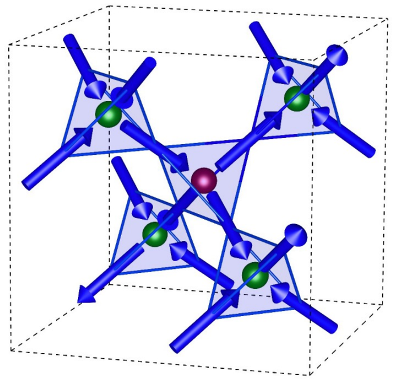
See also C. Castelnovo, R. Moessner, and S. L. Sondhi, PRB 84, 144435 (2011)

Monopole crystal

- **alternate – a 3in – 1out (3out-1in) vertex**

Brooks-Bartlett et al, PRX 4, 011007, 2014





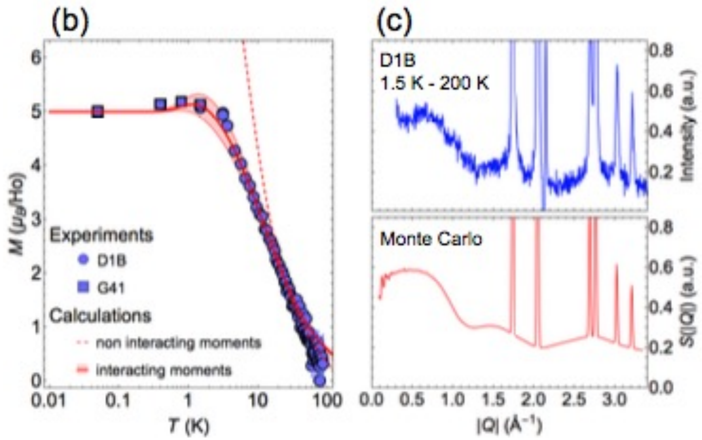
$$\left[M_{ij}\right] = \frac{m}{a}(-1,-1,-1,1) = \frac{m}{a}\left(-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2}\right) + \frac{m}{a}\left(-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},\frac{3}{2}\right)$$

$Ho_2Ir_2O_7$ and $Dy_2Ti_2O_7$

Lefrancois et al. Nature Communications 8, 209, 2017,
Cathelin et al. Phys. Rev. Research 2, 032073(R) 2020
Pearce, et al. Nat Commun 13, 444 (2022)

Ir_2 ions =>staggered monopole chemical potential

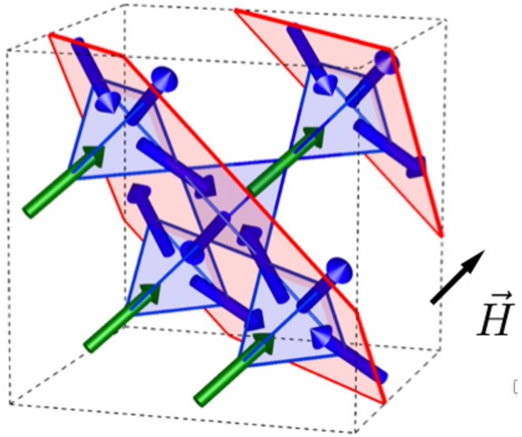
Saturated moment = ½ of total moment

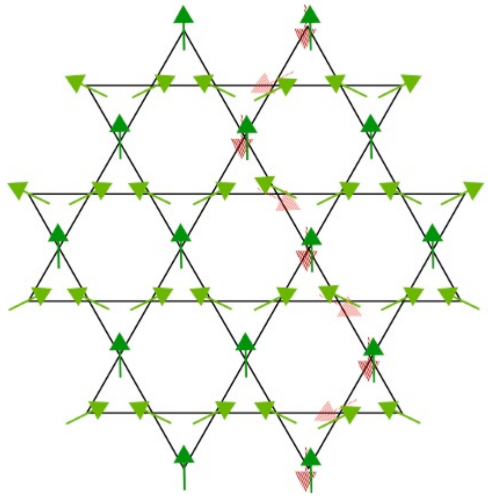


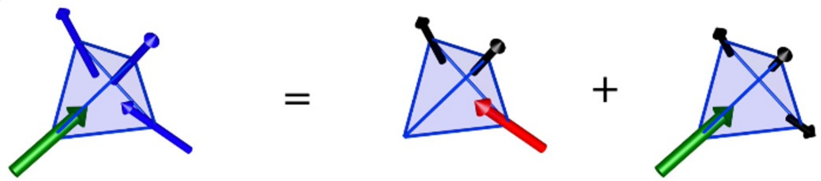
Full phase diagram

Raban et. al. , Phys. Rev. B 99, 224425, (2019)

Spin ice in [111] field => kagomé ice in consrained KII phase







$$[M_{\mathbf{r}\mu}]\,\frac{a}{m} = (-1,-1,1,1) = [0] + \left(0,\frac{2}{3},\frac{2}{3},-\frac{4}{3}\right) + \left(-1,\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$$

M= M_m + M_d + M_h

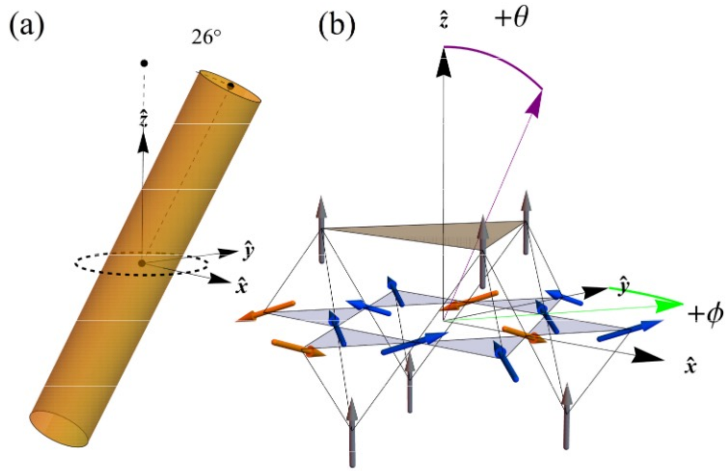
Projection $[M]_h \longrightarrow [M]_m^{2D}$ **2D charge from 3D harmonic contribution**

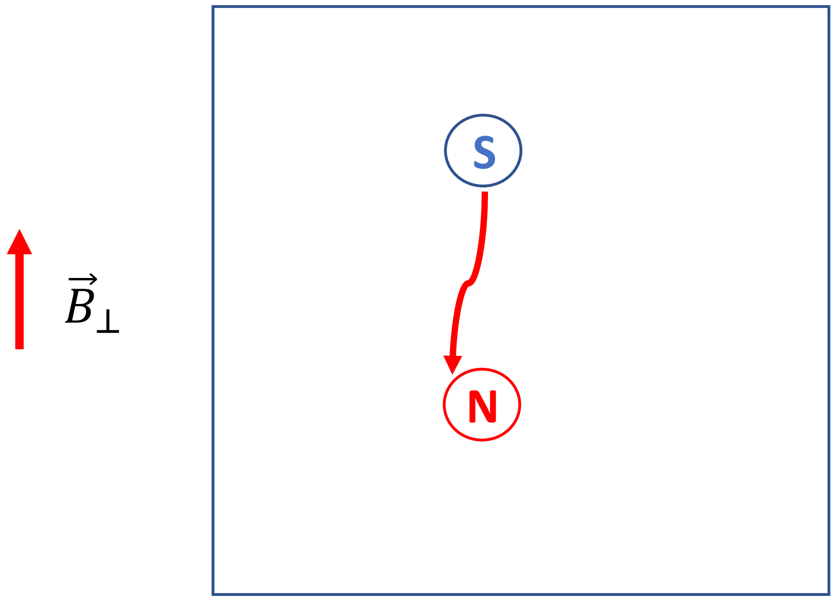
Moessner and S. L. Sondhi. Phys. Rev. B 68, 064411, 2003

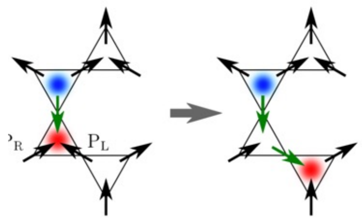
A. Turrini et al, PRB 105, 094403, 2022

T. Sakakibara, T. Tayama, Z. Hiroi, K. Matsuhira, and S. Takagi, Phys. Rev. Lett. 90, 207205 (2003).

Tilt away from [111] axis – Kasteleyn transition





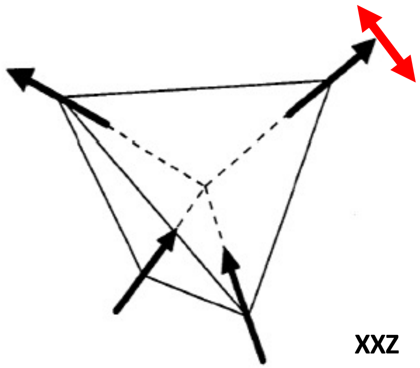


Confinement – deconfinement transition very similar to KT transition

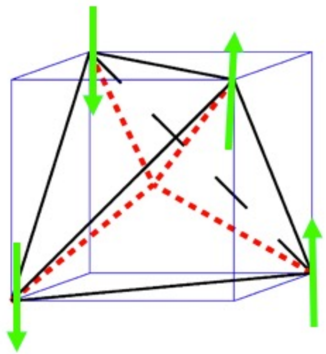
$U(r) \sim r$

$S(r) \sim r$

Quantum fluctuations and spin ice:

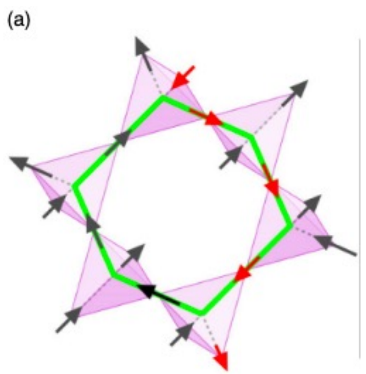


Transverse spin fluctuations
Create off-diagonal loop terms

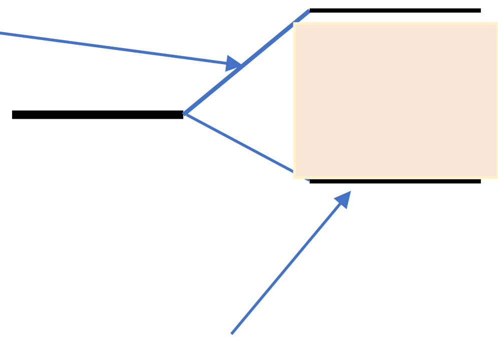


XXZ
M. Hermele, M. P. A. Fisher, and L. Balents, Phys. Rev. B 69, 064404 (2004)

Quantum spin ice
Benton, Sikora, Shannon, PRB, 86, 075154 (2012)
M J P Gingras and P A McClarty 2014 Rep. Prog. Phys. 77 056501
Lucile Savary and Leon Balents 2017 Rep. Prog. Phys. 80 016502



$$\mathcal{H}_{\text{tunneling}} = -g \sum_{\hexagon} [|\circlearrowright\rangle\langle\circlearrowleft| + |\circlearrowleft\rangle\langle\circlearrowright|],$$



Quantum spin liquid phase
Superposition of spin ice states
(transverse fragment)

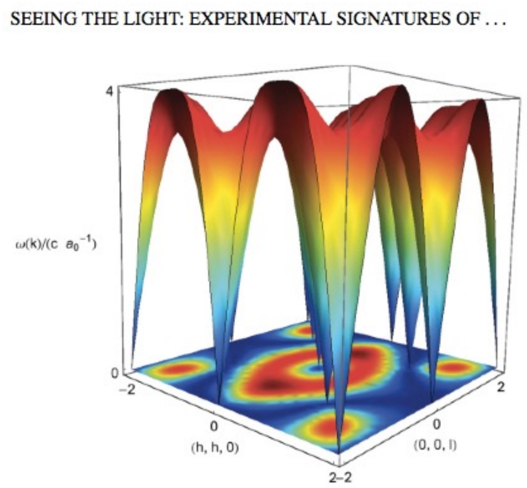
GS quantum singlet S=0

Quantum spin ice - Pocket QED

Benton, Sikora, Shannon, PRB, 86, 075154 (2012)

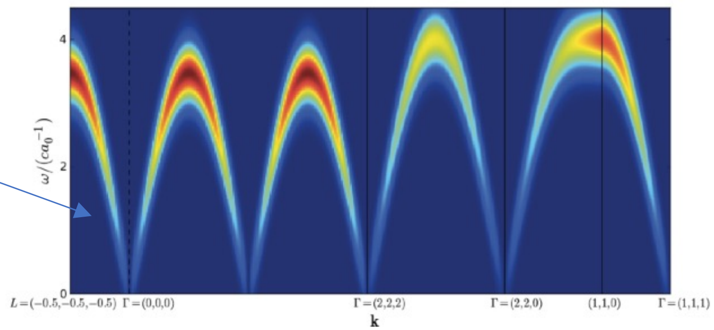
Coherent superposition of 2in-2out states opens band of low energy states – “photon” excitations

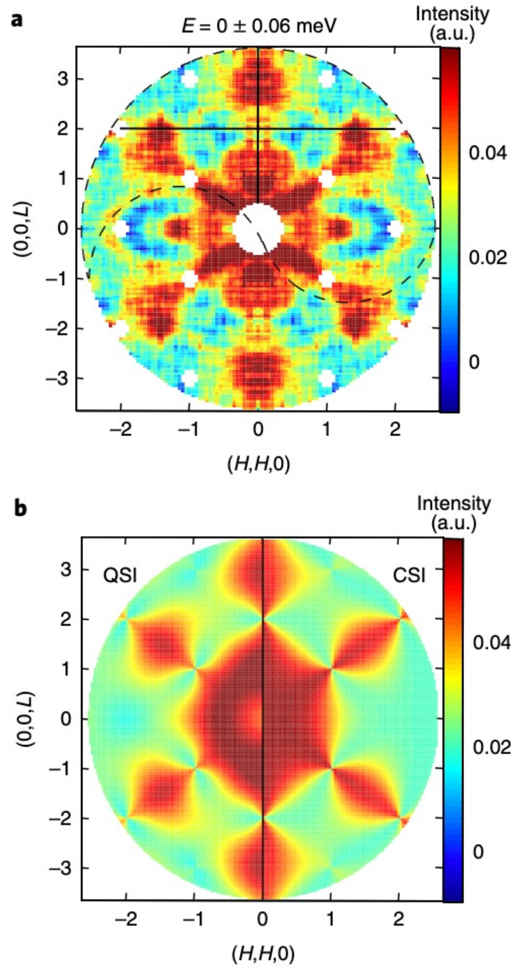
ε



$\varepsilon = \hbar c k$

Spin S=1





Best experimental candidate $Pr_2Hf_2O_7$

Sibille et. Al. Nat. Phys. 14, 711 (2018)

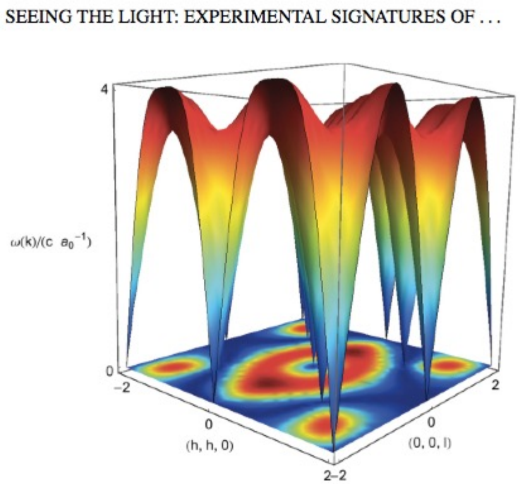
Quantum spin ice - Pocket QED

Benton, Sikora, Shannon, PRB, 86, 075154 (2012)

Coherent superposition of 2in-2out states opens band of low energy states – "photon" excitations

Phase fluctuations map to a second conjugate gauge field

ε



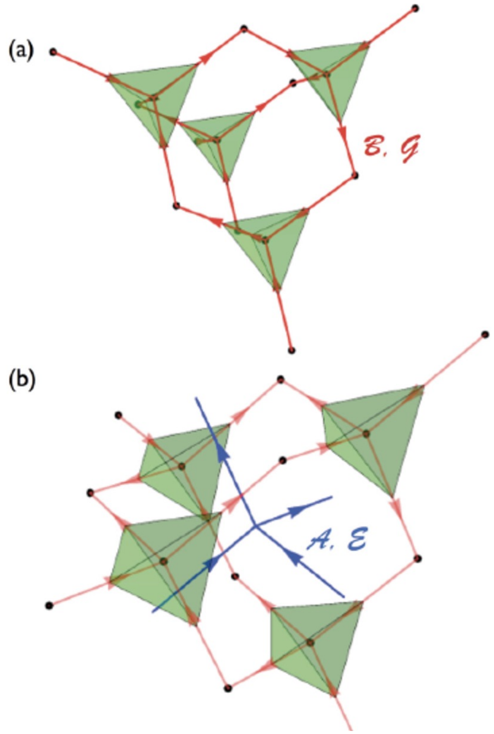
$\vec{B} = \vec{\nabla} \wedge \vec{A}$

Monopoles/spinons

$\vec{E} = -\vec{\nabla} \wedge \vec{G}$

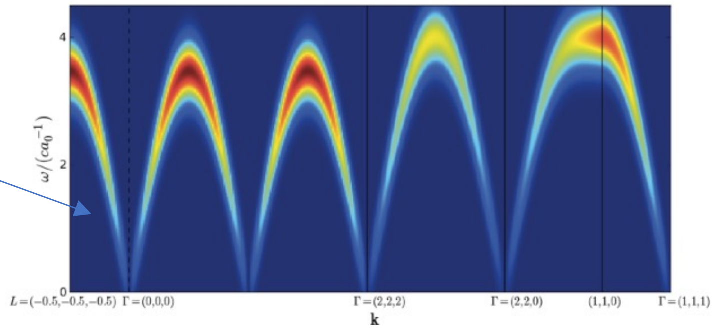
Electron/vison

Szabó and Castelnovo
Phys. Rev. B 100, 014417 (2019)



$\varepsilon = \hbar c k$

Spin S=1



Fine structure

Pace et. al. Phys. Rev. Lett. 127, 117

Best experimental candidate $Pr_2Hf_2O_7$

Sibille et. Al. Nat. Phys. 14, 711 (2018)