Transport and screening due to conformal anomaly in Dirac semimetals

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in collaboration with

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INSTITUT ORLÉANS • TOURS Mathématiques & Physique Théorique
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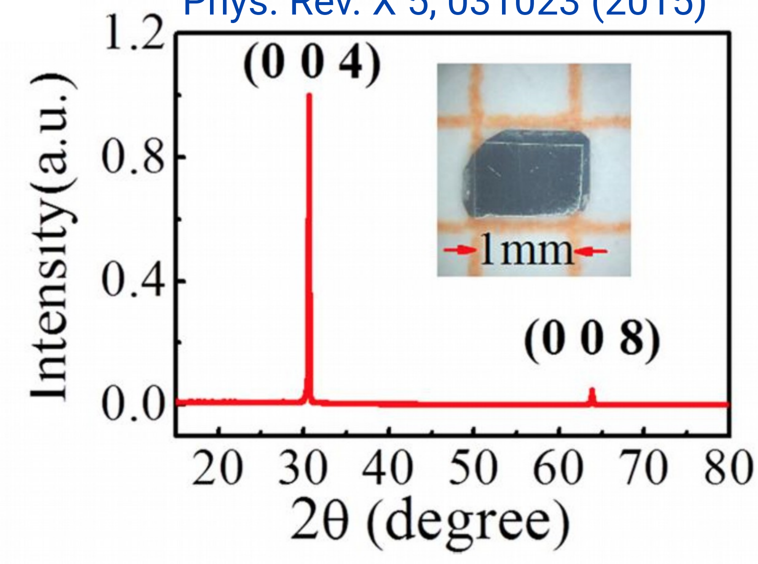
cnrs dépasser les frontières

Semimetals: real crystal materials

ArXiv:1503.01304

Phys. Rev. X 5, 031023 (2015)



Observation of the chiral anomaly induced negative magneto-resistance in 3D Weyl semi-metal TaAs

Xiaochun Huang[1,§], Lingxiao Zhao[1,§], Yujia Long[1], Peipei Wang[1], Dong Chen[1], Zhanhai Yang[1], Hui Liang[1], Mianqi Xue[1], Hongming Weng[1,2], Zhong Fang[1,2], Xi Dai[1,2] and Genfu Chen[1,2,*]

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Observation of Weyl nodes in TaAs

B. Q. Lv[1,2,*], N. Xu[2,3,*], H. M. Weng[1,4,*], J. Z. Ma[1,2], P. Richard[1,4], X. C. Huang[1], L. X. Zhao[1], G. F. Chen[1,4], C. Matt[2], F. Bisti[2], V. N. Strocov[2], J. Mesot[2,3,5], Z. Fang[1,4], X. Dai[1,4], T. Qian[1,§], M. Shi[2,§], and H. Ding[1,4,§]

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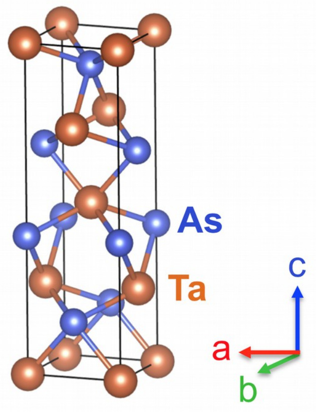
ArXiv:1503.09188

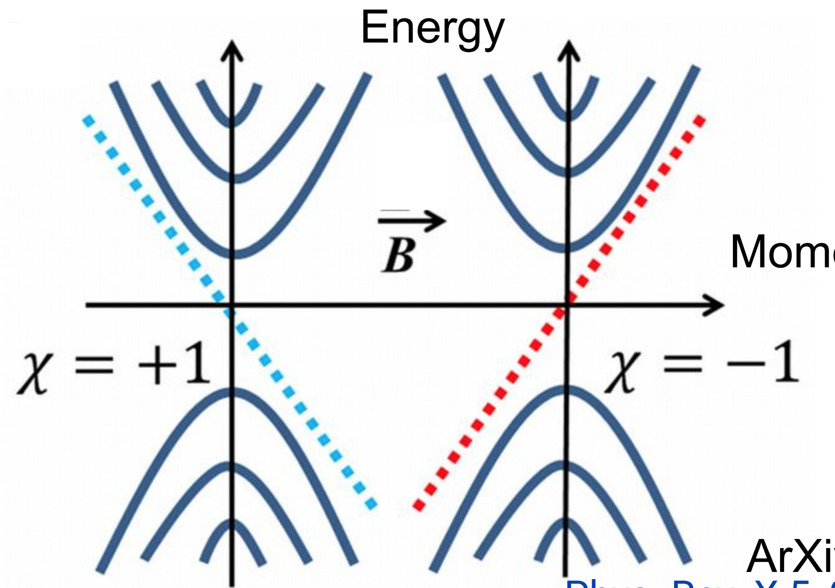
Nature Physics 11, 724-727 (2015)

- 1mm – 1 cm
- Good crystals



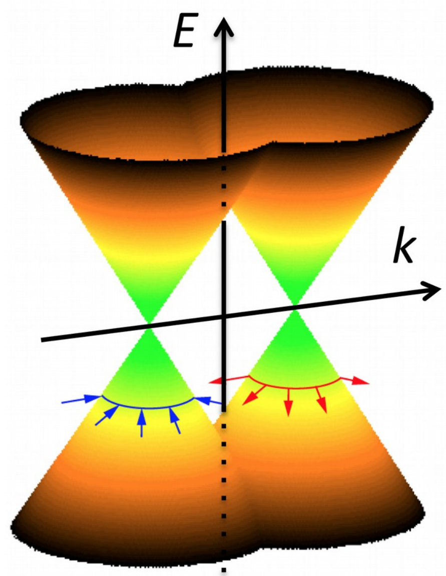
Chiral fermionic quasiparticles

Energy spectrum of a Weyl semimetal



TaAs

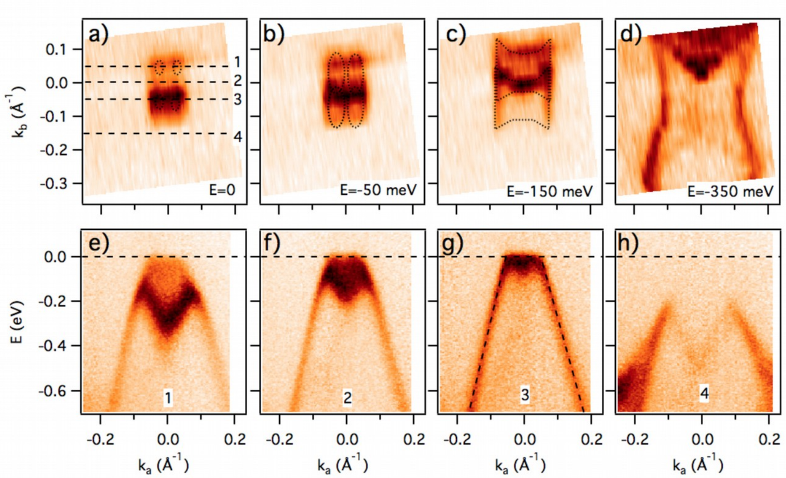
ArXiv:1503.09188

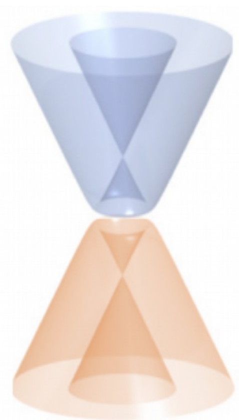


ArXiv:1503.01304

Phys. Rev. X 5, 031023 (2015)

Nature Physics 11, 724-727 (2015)





Dirac semimetal

$ZrTe_5$

ArXiv:1412.6543

Q. Li, D. E. Kharzeev et al., Nature Physics 12, 550 (2016)

Fermions and axial anomaly

Massless Dirac fermions

Covariant formulation

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} i \not{D} \psi$$

$$\not{D} = \gamma^\mu D_\mu \qquad D_\mu = \partial_\mu + ieA_\mu$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad \tilde{F}^{\mu\nu} = \tfrac{1}{2}\varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$$

Semimetals:

$$\bar{\psi}\left[i\gamma^0 \hbar \frac{\partial}{\partial t} + v_F \boldsymbol{\gamma}(i\hbar \boldsymbol{\nabla} - e\boldsymbol{A})\right]\psi$$

Currents

$$j_V^\mu = \bar{\psi}\gamma^\mu\psi, \qquad j_A^\mu = \bar{\psi}\gamma^5\gamma^\mu\psi$$

Vector — Axial

Axial anomaly

$$\partial_\mu j_A^\mu = \frac{e^2}{16\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

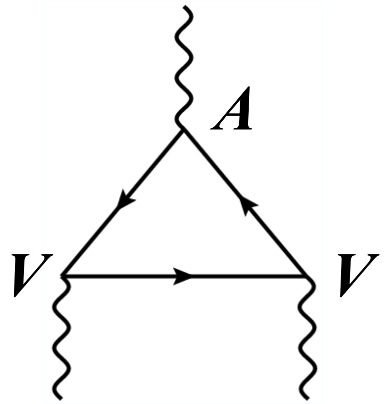
Usual (vector) chemical potential

Chiral (axial) chemical potential

Transport:

$$\boldsymbol{j}_A = \frac{\mu_V}{2\pi^2} e\boldsymbol{B}, \qquad \boldsymbol{j}_V = \frac{\mu_A}{2\pi^2} e\boldsymbol{B}$$

Chiral separation effect — Chiral magnetic effect



AVV diagram

Classical symmetries

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}i\not{D}\psi$$

Vector

$$\psi \to e^{i\omega_V}\psi$$

local/gauge symmetry

vector current is classically conserved

$$j^\mu_V = \bar{\psi}\gamma^\mu\psi \qquad \partial_\mu j^\mu_V = 0$$

Axial

$$\psi \to e^{i\omega_5\gamma^5}\psi$$

global symmetry (no axial gauge field)

axial current is classically conserved

$$j^\mu_A = \bar{\psi}\gamma^5\gamma^\mu\psi \qquad \partial_\mu j^\mu_A = 0$$

Conformal

global scale transformations

$$x \to \lambda^{-1}x, \qquad A_\mu \to \lambda A_\mu, \qquad \psi \to \lambda^{3/2}\psi$$

Dilatation current is classically conserved

$$j^\mu_D = T^{\mu\nu}x_\nu \qquad \partial_\mu j^\mu_D \equiv T^\mu_{\ \mu} \equiv 0$$

$$(T^\mu_\mu)_{\rm cl} \equiv 0$$

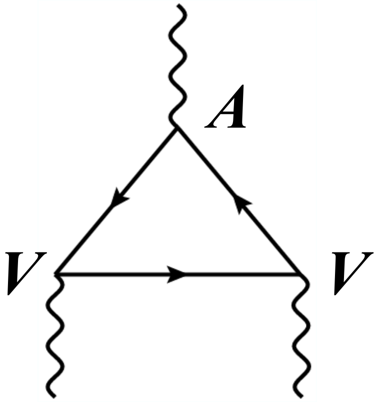
Energy-Momentum tensor

$$T^{\mu\nu} = -F^{\mu\alpha}F^\nu_{\ \alpha} + \frac{1}{4}\eta^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta} + \frac{i}{2}\bar{\psi}(\gamma^\mu D^\nu + \gamma^\nu D^\mu)\psi - \eta^{\mu\nu}\bar{\psi}i\not{D}\psi$$

Zoo of anomalies

(three out of six triangular vertices)

Axial

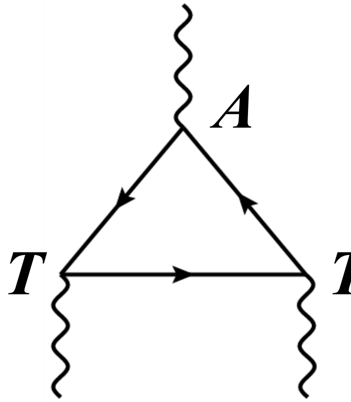


$$\partial_\mu j^\mu_A = \frac{e^2}{16\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$\tilde{F}^{\mu\nu} = \tfrac{1}{2}\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$$

Mixed

(axial-gravitational anomaly)

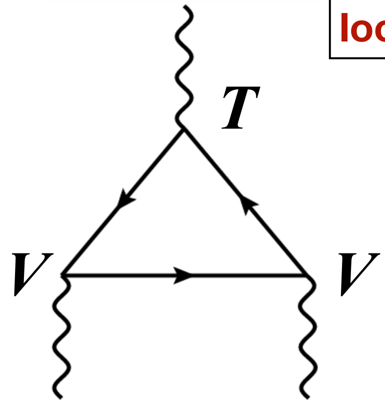


$$\partial_\mu j^\mu_A = -\frac{1}{384\pi^2} R_{\mu\nu\alpha\beta}\tilde{R}^{\mu\nu\alpha\beta}$$

$$\tilde{R}^{\mu\nu\alpha\beta} = \tfrac{1}{2}\varepsilon^{\mu\nu\gamma\lambda}R_{\gamma\lambda}{}^{\alpha\beta}$$

Conformal

not one-loop exact



$$\partial_\mu j^\mu_D = T^\alpha_\alpha$$

beta function

$$\langle T^\mu{}_\mu \rangle = \frac{\beta(e)}{2e} F_{\mu\nu}F^{\mu\nu}$$

Currents

$j^\mu_V = \bar{\psi}\gamma^\mu\psi,$ — **Vector**

$j^\mu_A = \bar{\psi}\gamma^5\gamma^\mu\psi$ — **Axial**

$j^\mu_D = T^{\mu\nu}x_\nu$ — **Dilatation**

Full list: AVV, ATT, TVV, TAA, AAA, TTT (not counting torsion!)

Energy-Momentum tensor

$$T^{\mu\nu} = -F^{\mu\alpha}F^\nu_\alpha + \frac{1}{4}\eta^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}$$

$$+\frac{i}{2}\bar{\psi}(\gamma^\mu D^\nu + \gamma^\nu D^\mu)\psi - \eta^{\mu\nu}\bar{\psi}i\not{D}\psi$$

Conformal anomaly and the beta function

Massless Dirac fermions $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} i \not{D} \psi$

are (classically) invariant under the global (scale) transformations:

$$x \to \lambda^{-1} x, \qquad A_\mu \to \lambda A_\mu, \qquad \psi \to \lambda^{3/2} \psi$$

The quantum theory generates an intrinsic scale due to a renormalization (in this particular case) of the electric charge:

$$\beta(e) = \frac{de(\mu)}{d \ln \underbrace{\mu}_{\text{renormalization scale}}}$$

In QED (for one Dirac fermion)

$$\beta_{\mathrm{QED}}^{1-\mathrm{loop}} = \frac{e^3}{12\pi^2}$$

→ **conformal symmetry is broken at the quantum level**

Quantum anomaly → anomalous transport

Axial anomaly

$$\partial_\mu j^\mu_A = \frac{e^2}{16\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

Transport

$$\boldsymbol{j}_A = \frac{\mu_V}{2\pi^2} e\boldsymbol{B}, \qquad \boldsymbol{j}_V = \frac{\mu_A}{2\pi^2} e\boldsymbol{B}$$

Chiral separation and chiral magnetic effects

topological = exact in one loop = interesting!

Mixed axial-gravitational anomaly

$$\partial_\mu j^\mu_A = -\frac{1}{384\pi^2} R_{\mu\nu\alpha\beta} \tilde{R}^{\mu\nu\alpha\beta}$$

Transport

$$\boldsymbol{j}_V = \frac{\mu_V \mu_A}{\pi^2} \boldsymbol{\Omega}, \qquad \boldsymbol{j}_A = \left(\frac{T^2}{6} + \frac{\mu_V^2 + \mu_A^2}{2\pi^2} \right) \boldsymbol{\Omega}$$

Thermal contribution to chiral vortical effects

topological = exact in one loop = interesting!

Conformal anomaly

$$\partial_\mu j^\mu_D = T^\alpha_\alpha \qquad \langle T^\mu_\mu \rangle = \frac{\beta(e)}{2e} F_{\mu\nu} F^{\mu\nu}$$

Transport?

not topological … not one-loop exact … not interesting?

Conformal anomaly →

Scale Electric Effect (SEE) and Scale Magnetic Effect (SME)

(Conformal Magnetic Effect = CME → interferes with Chiral Magnetic Effect … already taken, too late)

Oversimplified picture

Gravitational background: Weyl-transformed flat space

flat (Minkowski) metric

$$g_{\mu\nu}(x) = e^{2\tau(x)}\eta_{\mu\nu}$$

scale factor (arbitrary function of coordinates)

The conformal anomaly leads to scale electromagnetic effects:

$$\boxed{\langle T^{\mu}_{\ \mu} \rangle = \frac{\beta(e)}{2e} F_{\mu\nu}F^{\mu\nu}}$$

$$j^{\mu} \equiv \langle j^{\mu}_{V} \rangle = -\frac{2\beta(e)}{e} F^{\mu\nu}\partial_{\nu}\tau$$

"Generation of an electric current in a background of electromagnetic and gravitational fields"

Fine print: 1) natural in a linear-response theory
2) very unnatural (would-be-wrong) in general relativity
→ to be refined (and made less simple) later

M.Ch., PRL 117, 141601 (2016)

Scale electric effect (SEE)

Time-dependent background: $\tau = \tau(t)$

Metric: $g_{\mu\nu}(x) = e^{2\tau(x)}\eta_{\mu\nu}$

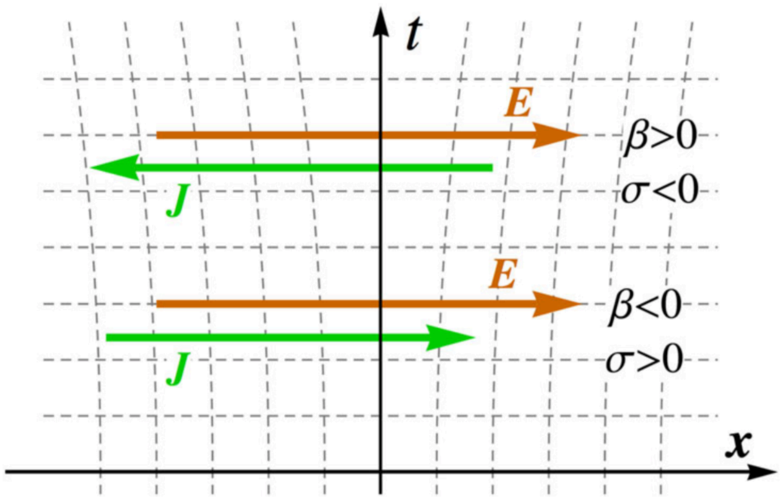
Scale Electric Effect:

$$\langle \boldsymbol{j}(t,\boldsymbol{x})\rangle_{\rm scale} = \sigma(t)\boldsymbol{E}(t,\boldsymbol{x}) \quad \text{for } \boldsymbol{\nabla}\tau = 0$$

Conformal conductivity:

$$\sigma(t,\boldsymbol{x}) = -\frac{2\beta(e)}{e}\frac{\partial\tau(t,\boldsymbol{x})}{\partial t}$$

Negative conductivity in an expanding space-time!



Independently obtained in the de Sitter spacetime (a version of the Schwinger effect, both for fermions and bosons)

→ T. Hayashinaka, T. Fujita, and J. Yokoyama, Fermionic Schwinger effect and induced current in de Sitter space, J. Cosmol. Astropart. Phys. 07 (2016) 010; T. Hayashinaka and J. Yokoyama, Point splitting renormalization of Schwinger induced current in de Sitter spacetime, J. Cosmol. Astropart. Phys. 07 (2016) 012.

→ T. Kobayashi and N. Afshordi, Schwinger effect in 4D de Sitter space and constraints on magnetogenesis in the early Universe, J. High Energy Phys. 10 (2014) 166.

Scale magnetic effect (SME)

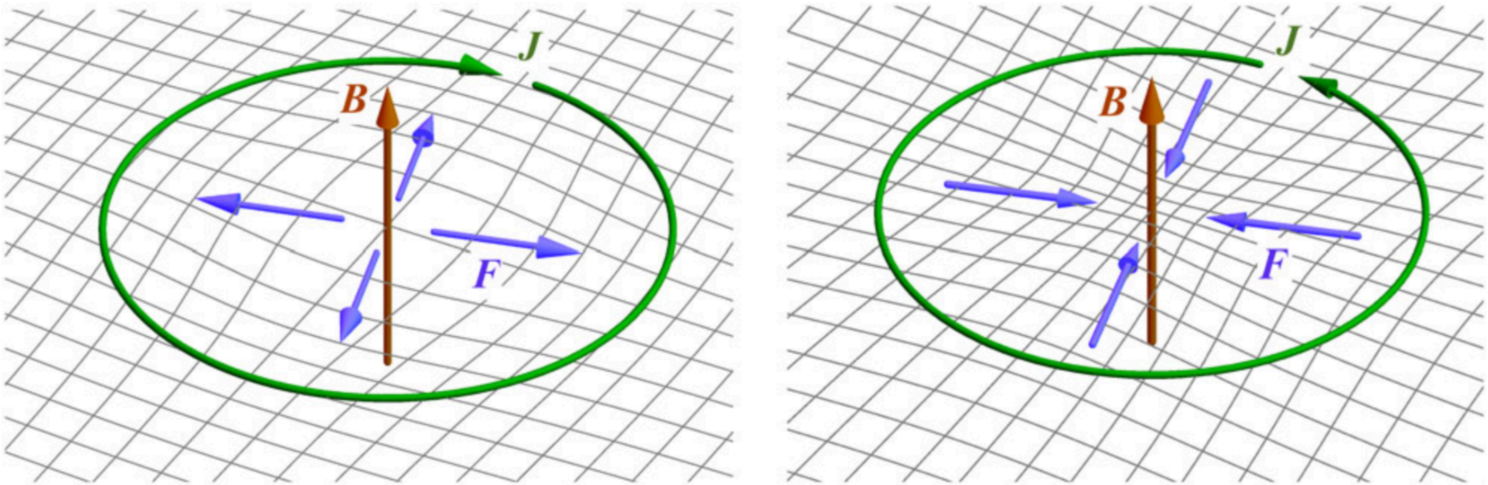
Space-dependent background: $\tau = \tau(\boldsymbol{x})$

Scale Magnetic Effect:

$$\langle \boldsymbol{j}(t,\boldsymbol{x})\rangle_{\text{scale}} = \boldsymbol{F}(\boldsymbol{x}) \times \boldsymbol{B}(t,\boldsymbol{x}) \quad \text{for } \partial_t \tau = 0$$

Gravitational deformation vector:

$$\boldsymbol{F}(t,\boldsymbol{x}) = \frac{2\beta(e)}{e} \boldsymbol{\nabla}\tau(t,\boldsymbol{x})$$



Distantly similar to Hall effects (no electric field, though).

Formal derivation of scale electromagnetic effects

Anomalous electric current from anomalous conformal action

$$J^\mu(x) = -\frac{1}{\sqrt{-g(x)}}\frac{\delta S_{\text{anom}}}{\delta A_\mu(x)}$$ ← variation with respect to the gauge field

$$= -\frac{1}{\sqrt{-g(x)}}\frac{\partial}{\partial x^\nu}\Big[\sqrt{-g(x)}\,F^{\mu\nu}(x)$$ ← electromagnetic field strength

$$\cdot \int d^4y \sqrt{-g(y)}\,\Delta_4^{-1}(x,y)\Big(E(y) - \frac{2}{3}\Box R(y)\Big)\Big]$$ ← space-time curvature enters here

Green's function for 4th order Paneitz conformal differential operator

Weak curvature, linear order:

$$J^\mu(x) = +\frac{e^2}{6\pi^2}F^{\mu\nu}(x)\partial_\nu\varphi(x)$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Minkowski / small perturbation

Effective scale factor:

$$\varphi(x) = \frac{1}{6}\int d^4y\,\Box^{-1}_{x,y}\left[\partial_\alpha\partial_\beta h^{\alpha\beta}(y) - \eta_{\alpha\beta}\Box h^{\alpha\beta}(y)\right]$$

→ for $g_{\mu\nu}(x) = e^{2\tau(x)}\eta_{\mu\nu}$

$$J^\mu = \frac{2\beta(e)}{e}F^{\mu\nu}\partial_\nu\tau$$

A. Cortijo, M. Vozmediano, M.Ch., PRL 120, 206601 (2018)

The SME in Dirac and Weyl semimetals

PHYSICAL REVIEW LETTERS **120**, 206601 (2018)

Generation of a Nernst Current from the Conformal Anomaly in Dirac and Weyl Semimetals

M. N. Chernodub,[1,2] Alberto Cortijo,[3] and María A. H. Vozmediano[3]

1.) A temperature gradient that drives system out of equilibrium may be mimicked by a gravitational potential (Luttinger):

$$\frac{1}{T}\boldsymbol{\nabla}T = -\frac{1}{c^2}\boldsymbol{\nabla}\Phi \qquad\qquad g_{00} = 1 + \frac{2\Phi}{c^2}$$

2.) In a magnetic-field background in a curved space-time, the conformal anomaly generates an electric current via the scale magnetic effect.

→ Conformal anomaly generates thermoelectric transport!

Anomalous electric current: $$J^\mu(x) = +\frac{e^2}{6\pi^2}F^{\mu\nu}(x)\partial_\nu\varphi(x)$$

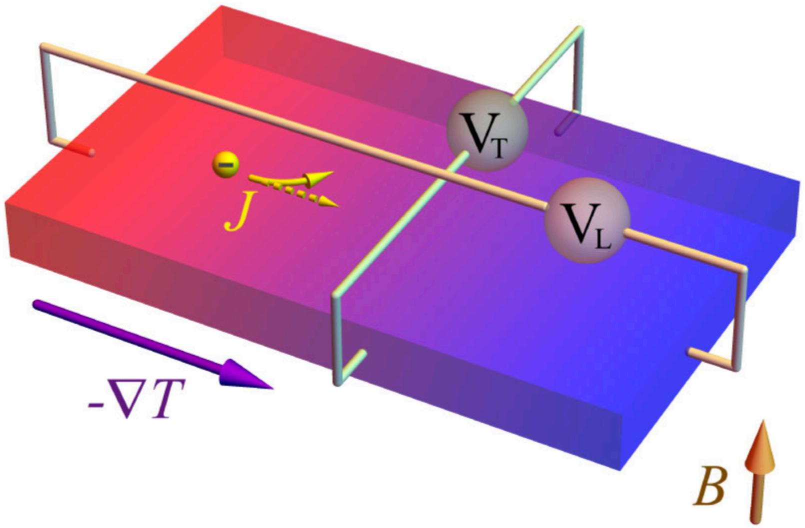
with the effective scale factor: $$\varphi(x) = \frac{1}{6}\int d^4y\,\Box^{-1}_{x,y}\left[\partial_\alpha\partial_\beta h^{\alpha\beta}(y) - \eta_{\alpha\beta}\Box h^{\alpha\beta}(y)\right]$$

(it will become a local quantity)

Nernst-Ettingshausen Effect in Weyl semimetals

Giant Nernst Effect due to SME

$$\boldsymbol{J} = \frac{e^2 v_F}{18\pi^2 T\hbar}\boldsymbol{B}\times\boldsymbol{\nabla}T$$



The electric current is proportional to the beta function (conformal anomaly!)

Longitudinal anomalous transport in Weyl semimetals:

$$J_i = \sigma_{ij}E_j + \mathcal{L}^{12}_{ir}(-\nabla_r T) = 0$$

$$E_j = \rho_{ji}\mathcal{L}^{12}_{ir}(-\nabla_r T)$$

$$\mathcal{L}^{12}_{21} = \frac{e^2 v_F B_3}{18\pi^2\hbar T}$$

$$S_{11} \equiv \frac{E_1}{B_3\nabla_1 T} = \frac{\rho_{12}\mathcal{L}^{12}_{21}}{B_3} \sim \frac{v_F}{9|\boldsymbol{b}|T}$$

Estimations for an undoped Weyl semimetal ($v_F \sim 10^5$ m/s, $T \sim 10$ K, $|2\boldsymbol{b}| \sim 0.3$ Å^{-1}**)**

$$S_{11}/T \sim 0.6\ \mu\mathrm{V}/\mathrm{T\,K}^{-2}$$

Accessible experimentally!

works via the anomalous Hall current

$$\boldsymbol{J} = \frac{e^2}{2\pi^2\hbar}\boldsymbol{b}\times\boldsymbol{E}.$$

A. Cortijo, M. A. H. Vozmediano, M.Ch., PRL 120, 206601 (2018)

Remarks on the Scale Magnetic Effect (SME)

– Appears due to conformal anomaly $\langle T^{\mu}_{\ \mu} \rangle = \frac{\beta(e)}{2e} F_{\mu\nu} F^{\mu\nu}$

– Bulk phenomenon, works at zero chemical potential

– Leads to a giant Nernst effect in Dirac/Weyl semimetals

$$\boldsymbol{J} = \frac{e^2 v_F}{18\pi^2 T \hbar} \boldsymbol{B} \times \boldsymbol{\nabla} T$$

– Is not related to axial or axial-gravitational anomalies

– Strength is given by the beta function $\beta(e) = \frac{de(\mu)}{d \ln \mu}$

– Universal: works both in fermionic and bosonic systems

Physical manifestation of the Scale Electric Effect:

Negative thermal inertia of the electric conductance in Dirac and Weyl semimetals

– Bulk phenomenon, works at zero chemical potential

– Leads to a giant Nernst effect in Dirac/Weyl semimetals

– Is not related to axial or axial-gravitational anomalies

– Strength is given by the beta function

$$\beta(e) = \frac{de(\mu)}{d\ln\mu}$$

– Universal: works both in fermionic and bosonic systems

Scale Electric Effect: negative conductivity in time-dependent backgrounds

Conformal anomaly and transport effects at the edge

What is about the boundaries?

electric current

beta function

normal to the boundary

$$J^\mu = -\frac{2c\beta_e}{e\hbar}\frac{F^{\mu\nu}n_\nu}{x}$$

distance to the reflective boundary

D.M. McAvity, H. Osborn, Class. Quantum Gravity 8, 603 (1991).
C.-S. Chu and R.-X. Miao, JHEP 07, 005 (2018), PRL 121, 251602 (2018).

Scale Magnetic Effect at the Edge (SMEE):

Electric current along the edge due to tangential magnetic field

$$\boldsymbol{j}(\boldsymbol{x}) = -f(\boldsymbol{x})\,\boldsymbol{n}\times\boldsymbol{B}$$

$$f(\boldsymbol{x}) = \frac{2\beta(e)}{e^2}\frac{1}{x_\perp}$$

– Effect due to conformal anomaly
– No topology (Berry, Chern etc)

diverges at the boundary!

Scale electric effect at the edge: conformal screening

Screening of electrostatic field in metals:

$$E(x) \sim E(0) e^{-x/\lambda}$$

Fermi momentum

$$p_F = (2\pi^2 n)^{1/3} \hbar$$

Density of carriers n

Screening lengths:

$$\lambda_{\mathrm{D}} = \sqrt{\frac{\varepsilon_0 k_B T}{n e^2}}, \qquad \lambda_{\mathrm{FT}} = \sqrt{\frac{\varepsilon_0 \pi^2 \hbar^3}{m e^2 p_F}}$$

Debye **Fermi-Thomas**

What if the medium is totally conformal and possess no dimensionful parameters?

For example, take a Dirac semimetal at particle-hole symmetric point.

– We have the mobile carriers (massless fermionic quasiparticles)

– Classically, there is no dimensionful scale.

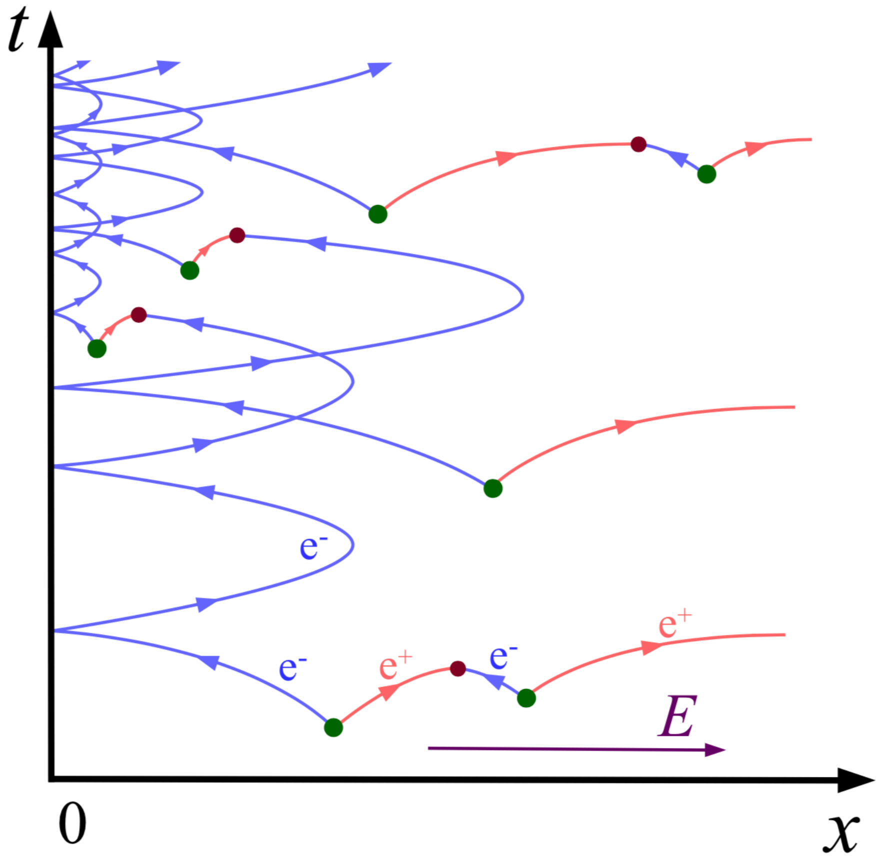
No quantity to construct the screening length from!

– No classical scale → no screening?

Scale electric effect at the edge

$$J^\mu = -\frac{2c\beta_e}{e\hbar}\frac{F^{\mu\nu}n_\nu}{x} \longrightarrow \rho = -\frac{2\beta_e}{e\hbar}\frac{\boldsymbol{nE}}{x}$$

the density of the electric charge accumulated at the boundary



Physics: the screening is due to the Schwinger effect (skipping orbits in time)

Works efficiently due to the absence of a mass gap

Generated by the conformal anomaly!
(proportional to the beta function)

Mechanism in semimetals: creation of electron-hole pairs in the presence of a uniform electric field (the Zener effect)

Scale electric effect at the edge of a semimetal

Consider QED:

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \sum_{a=1}^{N_f} \bar{\psi}_a i \gamma^\mu D_\mu \psi_a$$

Charge density due to conformal anomaly: $\rho = -\frac{2\beta_e}{e\hbar} \frac{\boldsymbol{n}\boldsymbol{E}}{x}$

Solve the Maxwell equation $\partial_x E_x(x) = \frac{1}{\varepsilon_0} \rho(x)$

At the boundary the conformal screening is polynomial:

$E_x(x) = \dfrac{C}{x^\nu}$	$\rho(x) = -\dfrac{C\varepsilon_0 \nu}{x^{1+\nu}}$	$\phi(x) = \phi_0 - \dfrac{C x^{1-\nu}}{1-\nu}$
Electric field	**Charge density**	**Electrostatic potential**

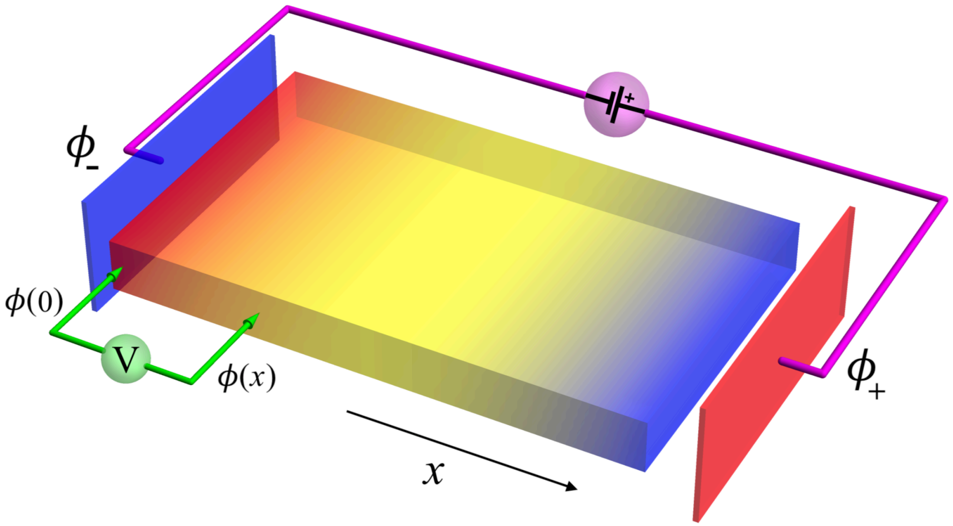
Conformal screening exponent: $\nu = \frac{2\beta_e}{e\hbar\varepsilon_0}$

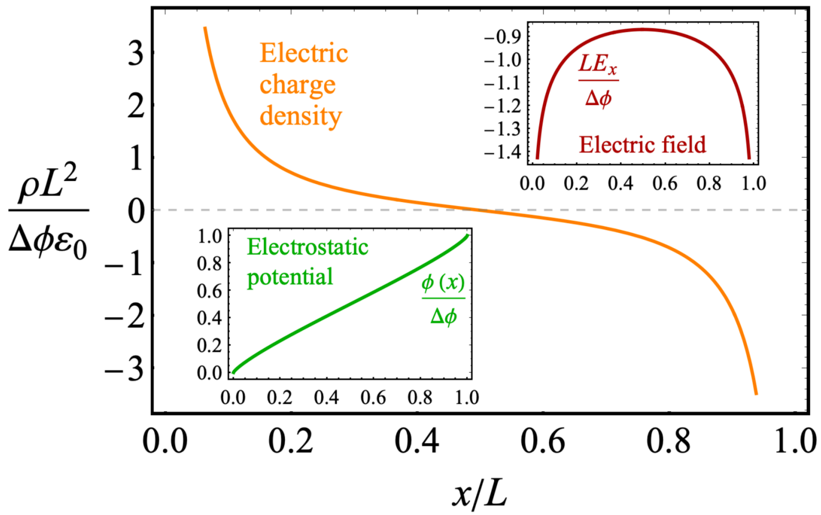
Scale electric effect at the edge

Conformal exponent in a Dirac semimetal: $$\nu = \frac{e^2}{6\pi^2 \hbar v_F \varepsilon\varepsilon_0} = \frac{2\beta_e}{e c \hbar \varepsilon_0}$$

Particle density in a finite sample with two boundaries:

$$\rho(x) = \frac{\Delta\phi}{L^2}\varepsilon_0 \nu h(\nu) \left(1 - \frac{2x}{L}\right) \left[\frac{x}{L}\left(1 - \frac{x}{L}\right)\right]^{-1-\nu}$$





Direct measurement of the beta function. Indirect evidence of the Schwinger effect.

Phys. Rev. Research 1, 032002 (2019)

M. A. H. Vozmediano, M.Ch., arXiv:1902.02694

Accessible experimentally

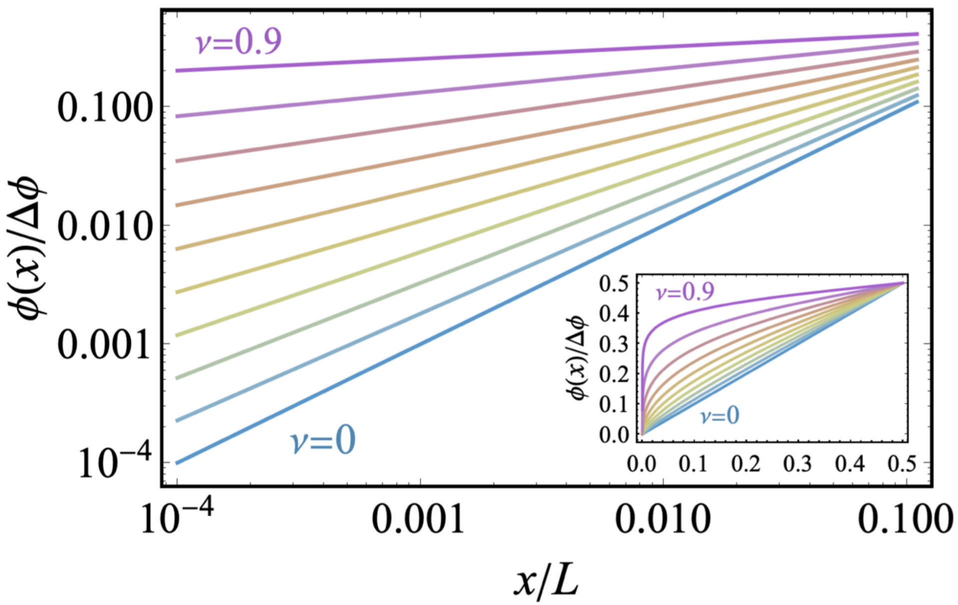
- direct measurement of the beta function
  associated with the renormalization of the electric charge
  (never done in solid state)
- evidence of the elusive Schwinger effect
  (particle-antiparticle production by electric field)

Conformal exponent in a Dirac semimetal: $$\nu = \frac{e^2}{6\pi^2 \hbar v_F \varepsilon \varepsilon_0}$$

In typical Dirac/Weyl materials $v_F \sim 10^{-3} c$ and $\varepsilon \sim 10$

→ large, experimentally accessible conformal exponent: $\nu \sim 10^{-1}$

Electrostatic screening potential vs. distance from the boundary of a Dirac material (at the Lifshitz point)



Summary

Conformal anomaly leads to a number of new transport effects:
— in the bulk (unbounded systems)
— at reflective boundaries (edges) of bounded systems

Currents are proportional to the beta function.
Accessible experimentally in Dirac and Weyl semimetals.

Scale electric effect:

- — negative conductivity in expanding systems
- — logarithmic screening of electrostatic fields
- — particle creation via the Schwinger effect

Scale magnetic effect:

- —thermoelectric transport, Nernst effects
- —edge currents in the absence of matter